Statistics 406 Problem Set 1

Due in lab, Tuesday September 18

1. You can get normal tail probabilities in R using the function `pnorm`, i.e. if Z has a standard normal distribution, then $P(Z \leq t)$ can be obtained in R using `pnorm(t)`. Use R to generate a table as follows:

	Exact	Simulation estimate
P$(-3 \leq Z < -2.5)$		
P$(-2.5 \leq Z < -2)$		
P$(-2 \leq Z < -1.5)$		
P$(-1.5 \leq Z < -1)$		
P$(-1 \leq Z < -0.5)$		
P$(-0.5 \leq Z < 0)$		
P$(0 \leq Z < 0.5)$		
P$(0.5 \leq Z < 1)$		
P$(1 \leq Z < 1.5)$		
P$(1.5 \leq Z < 2)$		
P$(2 \leq Z < 2.5)$		
P$(2.5 \leq Z < 3)$		

Solution:

```
## Storage for the simulation results.
S = NULL

## Storage for the exact results.
E = NULL

## A large number of simulated standard normal draws.
Z = rnorm(1e5)

## The lower bound of the current interval.
T = -3

## Cycle through the rows of the table.
for (k in 1:12) {
  S[k] = mean( (Z >= T) & (Z < T+0.5) )
  E[k] = pnorm(T+0.5) - pnorm(T)
  T = T+0.5
}
```

2. Suppose U_1 and U_2 are generated uniformly, and someone claims that the random value

$$Z = \sqrt{-2 \log U_1} \cos(2\pi U_2)$$

has a standard normal distribution. Using simulation, fill in the table from problem 1 using this definition of Z, and assess whether it appears to behave like a normally distributed value.

Solution:

Using the following code, you'll see that the simulated probabilities for Z agree very closely with the standard normal distribution. It is a fact that Z is exactly standard normal.

```
## Storage for the simulation results.
S = NULL

## A large number of simulated standard normal draws.
U1 = runif(1e5)
U2 = runif(1e5)
Z = sqrt(-2*log(U1)) * cos(2*pi*U2)

## The lower bound of the current interval.
T = -3

## Cycle through the rows of the table.
for (k in 1:12) {
  S[k] = mean( (Z >= T) & (Z < T+0.5) )
  T = T+0.5
}
```

3. Suppose we flip a fair coin until we get the first head. Let N be the number of flips that are made, including the flip that is a head. The exact probabilities for N are given by:

$$P(N = n) = 1/2^n$$

Using R, carry out a simulation to estimate the probability that the first head occurs within 4 flips. Compare your finding to the exact probability, using the fact that

$$P(N \leq n) = P(N = 1) + P(N = 2) + \cdots + P(N = n).$$

Solution:

The exact probability is 0.9375. You'll see that the simulation results agree very well with the exact value.

```
## The number of simulations.
nsim = 1e4

## Keep track of how many times the first head occurs within four flips.
N4 = 0

## Simulation loop.
for (r in 1:nsim) {

  ## Simulate one sequence of coin flips up to the first head.
  N = 0
  while (1) {

    ## Count the flips.
    N = N+1

    ## The result of this flipe (1=heads)
    F = runif(1) < 0.5

    ## Stop flipping if we get a head.
    if (F==TRUE) { break }
  }

  ## Keep track of how often we got the head within four flips.
  if (N<=4) { N4 = N4+1 }
}

## The estimated probability.
p_est = N4/nsim

## The exact probability.
p_exact = 1/2 + 1/4 + 1/8 + 1/16
```