Additional HW3 problem.

The number of eggs laid by an insect is a Poisson(λ) random variable. Each egg that is laid hatches independently of the others with probability p.

Let X denote the total number of eggs laid, X_1 the number that hatch and X_2 the number that don't.

Show that X_1 and X_2 are independent random variables in the sense that:

$$P(X_1 = \ell, \ X_2 = m) = P(X_1 = \ell) \, P(X_2 = m)$$