Midterm 1, Winter 06

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Announcement: The exam carries 30 points but the maximum you can score is 25.

(1) If X and Y are two uncorrelated random variables, are they necessarily independent? On the other hand, if they are independent, are they necessarily uncorrelated? Justify your answers. (6)

(2) Let X and Y be independent standard normal random variables. Set $U = (3X + 4Y)/5$ and $V = (4X - 3Y)/5$.

(i) Find the joint density of (U, V) and also the marginal densities. (8)

(ii) Show that $U^2 + V^2$ is distributed like an exponential random variable with parameter $1/2$. (4)

(3) Two helicopters land independently of each other on the plane. Let (X_1, X_2) denote the co-ordinates of the first and (Y_1, Y_2) denote the co-ordinates of the second. It may be assumed that X_1 and X_2 are independent standard normal variables, and likewise for Y_1 and Y_2.

(i) Show that, on an average, the locations of the helicopters are connected by a line with the origin as midpoint. (6)

(ii) Let D denote the distance between the helicopters. Show that $E(D^2) = 4$. Also find the chance that D is larger than x, where $x > 0$. (6)