Midterm 1: Stat 426.

Moulinath Banerjee

University of Michigan

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Announcement: The total number of points is 30 but the maximum you can score is 25.

(1) (a) Suppose that the radius of a circle is a random variable having the following probability density function.

$$f(x) = \frac{1}{8}(3x+1), \quad 0 < x < 2$$

and 0 otherwise. Determine the probability density function of the area of the circle. (6)

(2) Let (X, Y) be uniformly distributed inside the ellipse given by $x^2/a^2 + y^2/b^2 = 1$. Thus, the joint density of (X, Y) is:

$$f(x,y) = \frac{1}{\pi\, a\, b}\, 1\left\{\frac{x^2}{a^2} + \frac{y^2}{b^2} < 1\right\}.$$

where recall that $1\{x^2/a^2 + y^2/b^2 < 1\}$ is the indicator function that is 1 if $x^2/a^2 + y^2/b^2 < 1$ and 0 otherwise.

(a) Let $(U, V) = (\epsilon_1 X, \epsilon_2 Y)$ where ϵ_1 and ϵ_2 are either 1 or -1. Show that (U, V) has the same joint distribution as (X, Y).

(b) Are X and Y independent? Are they uncorrelated? Justify your answers. (6+6)

(3) (a) Let $X_1, X_2, \ldots, X_p$ be i.i.d. Exponential(λ) random variables. Show that $2\lambda X_1 + 2\lambda X_2 + \ldots + 2\lambda X_p$ follows a χ^2_{2p} distribution. (**Hint:** How is $2\lambda X_1$ distributed?)

(b) Show (using definitions) that (i) If T follows a t–distribution on n degrees of freedom, then T^2 follows an $F_{1,n}$ distribution. (ii) If H follows an $F_{m,n}$ distribution, then $1/H$ follows an $F_{n,m}$ distribution. (6 + 6)