Homework 5.

Math | Stat 425

1. Pages 249 – 250 16, 20, 21, 26, 27

2. A random variable V is said to have a distribution symmetric about 0 if the distribution of V is the same as that of $-V$. Let V be a continuous random variable with density f.

   (a) Show that V is symmetric about 0 if and only if $f(t) = f(-t)$ for every t.

   (b) Show ~~that~~ $EV = 0$. What is $E(V^{2m+1})$ for an arbitrary integer m?

3. A line is drawn through a point in a random direction. Let $(-1, 0)$ be the co-ordinates of the point and let $(0, Y)$ denote the point at which the line cuts the vertical axis. Show ~~that~~ Y has the standard Cauchy density i.e: $f_Y(y) = \frac{1}{\pi} \cdot \frac{1}{1+y^2}$, $y \in \mathbb{R}$.

4. Show ~~that~~ if Y follows the Cauchy density, so does $\frac{1}{Y}$.

5. Under appropriate assumptions, use the Change of Variable Theorem to show ~~that~~: (a) If X is a random variable with density f and cdf F, $F(X) \sim \text{Uniform}(0, 1)$.

   (b) Let $\tilde{X} = F^{-1}(U)$ where U is Uniform$(0,1)$. Then $\tilde{X}$ has density f and is distributed like X.

6. (a) Let $Z \sim N(0,1)$. Using that $E(Z^2) = 1$ and the relation: $E(X^{2k}) = (2k-1)E(X^{2k-2})$ which you need to prove, find a general expression for $E(X^{2k})$. What is $E(X^{2k+1})$, k arbitrary?

(b) Let T be an exponential(λ) random variable, whose value has been observed. Independently of the process generating T (say T is the time to arrival of a bus at a bus-stop), I flip a coin. If the coin lands H, I transform T to $-T$. Otherwise I keep T intact. Assume a fair coin. Find the density of the random variable produced after the coin toss.