Exam 2: Statistics 426.

13 Dec, 2006

Announcement: The exam carries 40 points but the maximum possible score is 35.

(1) Suppose that $X_1, X_2, \ldots, X_n$ are i.i.d Unif$(-\theta^{-2}, \theta^{-2})$ for some $\theta > 0$. Find a MOM estimate for θ and the MLE of θ based on the X_i's. (8)

(2) Consider i.i.d. data $\{X_i, Y_i\}_{i=1}^n$ where X_i is the blood–pressure of indiviudal i before taking a drug and Y_i is the blood–pressure of the same individual after being on the drug. It may be assumed that $X_i \sim N(\mu_1, \sigma^2)$ and $Y_i \sim N(\mu_2, \sigma^2)$ for some unknown σ and that the correlation between X_i and Y_i is some (unknown) positive fraction ρ. Of interest is the difference in blood–pressure $\mu_1 - \mu_2$.

(i) How can you use the difference in observed blood pressures: the $X_i - Y_i$'s, to construct a level $1 - \alpha$ confidence interval for the quantity of interest?

(ii) Suppose now that the doctor wants a lower confidence limit on the difference of mean blood–pressures – i.e. a number a such that the chance that the difference in mean blood pressures is at least a is $1 - \alpha$. Can you provide such a number based on your data?

(iii) Suppose that the drug is strongly believed to lower blood pressure and the doctor wants you to incorporate this information in your analysis. Suggest modified confidence intervals for $\mu_1 - \mu_2$ that account for this. (10)

(3) Suppose that $X_1, X_2, \ldots, X_n$ are i.i.d. with density $f(x, \theta) = (\theta + 1)\, x^\theta$, $0 < x < 1$ and $\theta > 0$. Use the limit distribution of the MLE of θ and the method of variance stabilizing transformations to construct an approximate level $1 - \alpha$ confidence interval for θ. (7)

(4) Suppose that $Y_1, Y_2, \ldots, Y_n$ are i.i.d. Poisson(θ). However, the original Y_i's get lost and information is only available as to whether each Y_i was zero or non–zero. Thus, the available data are $X_1, X_2, \ldots, X_n$ where $X_i = 1(Y_i = 0)$. Compute the MLE of θ based on the X_i's. How does this compare to the MLE of θ based on the Y_i's (assuming the Y_i's are available) in large samples? A good qualitative answer works for this last part. (7)

(5) Particles are emitted by a radioactive source one by one with the time gap between two successive emissions being distributed exponentially with mean β. Thus, the time to emission of the 1st particle is an exponential random variable with mean β, the time that elapses between the emission of the 1st and that of the 2nd is also exponential with mean β (and independent of the first variable) and so on. A physicist measures the first n inter–emission times but then loses his/her data, only managing to remember the total time that elapses till the emission of the last observed particle. How would he/she find a confidence interval for the mean inter–emission time based on this data? (8)