

Midterm 1: Stat 426.

Moulinath Banerjee

University of Michigan

February 14, 2002

Announcement: The total number of points is 25 but the maximum you can score is 20.

- 1. Alvie Singer lives at O in the diagram below and has four friends who live at A, B, C and D. One day Alvie decides to go visiting, so he tosses a fair coin twice to decide which of the four friends to visit. Once at a friend's house, he will either return home or else proceed to one of the two adjacent houses (such as O, A or C when at B) with each of the three possibilities having probability $1/3$. In this way, Alvie continues to visit friends till he returns home. Let X be the number of times that Alvie visits a friend. Find the probability mass function of X , i.e. $P(X = j)$ for $j = 0, 1, 2, \dots$ etc. (7 points)

- 2. Let X_1 and X_2 be independent $N(0, 1)$ random variables. Let

$$W_1 = \frac{X_1 + X_2}{\sqrt{2}} \quad \text{and} \quad W_2 = \frac{X_1 - X_2}{\sqrt{2}}.$$

(i) Using the Jacobian Theorem (or otherwise) show that W_1 and W_2 are independent $N(0, 1)$ random variables.

(ii) How is W_2^2 related to $(X_1 - \bar{X})^2 + (X_2 - \bar{X})^2$? (\bar{X} is just the average of X_1 and X_2)

(iii) Deduce from (ii) that $(X_1 - \bar{X})^2 + (X_2 - \bar{X})^2$ follows a χ_1^2 distribution. Also, show that it is independent of \bar{X} .

(iv) Identify the distribution of $(X_1 + X_2) / |X_1 - X_2|$. You do not need to work out the density; in fact, don't go down that track. Just identify it as a distribution you know/have seen. (8 + 3 + 4 + 3 = 18 points)