Midterm 1: Stat 426.

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Announcement: The total number of points is 25 but the maximum you can score is 20.

(1) Let X be an Exponential(1) random variable and define $Y = [X] + 1$ where $[X]$ is the largest integer not exceeding X. Then Y is a discrete random variable.

Compute the probability mass function of Y. Can you identify the distribution of Y as something you have seen before? What is $E(Y)$? (7 points)

(2) A, B and C play the following game: A fair die is rolled and A wins if 1 or 2 turn up, B wins if 3 or 4 turn up and C wins if 5 or 6 turn up. They keep on playing till B wins for the first time. Find the chance that they have to keep playing k games till B wins. What is the average number of games that they need to play? (8 points)

(3) Let X and Y be independent Exponential (1) random variables. Let $U = X$ and $V = X/Y$. Compute the joint density of (U, V) and hence, find the marginal densities of U and V. (10 points)