Midterm 2: Stat 426.

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December 3, 2004

Announcement: The total number of points is 23 but the maximum you can score is 20.

(1) Decide on whether the following statements are TRUE or FALSE with a brief justification.

(a) A confidence interval of level $1-\alpha$ for μ based on i.i.d. data $X_1, X_2, \ldots, X_n$ following $N(\mu, \sigma^2)$ will generally be larger than a confidence interval of level $1-\gamma$, based on the same data, if $\alpha > \gamma$. $(0 < \alpha, \gamma < 1)$.

(b) Constructing a confidence interval with confidence level 1 is the best thing to do, since we are then guaranteed to trap the true parameter value in our interval.

(c) The MLE can always be obtained by differentiating the log likelihood function.

(d) In a coin tossing experiment, the sample proportion of heads approaches the true underlying probability of heads as the number of tosses gets larger and larger

(e) Consider i.i.d. data $X_1, X_2, \ldots, X_n$ following $N(\mu, \sigma^2)$ with μ known but σ^2 unknown. In this case, $\sum_{i=1}^{n} (X_i - \mu)^2/\sigma^2$ is a valid pivot and can be used to find a confidence interval for σ^2. ($2 \times 5 = 10$ points).

(2) Consider the model $Y_i = \beta\, X_i + \epsilon_i$ for $i = 1, 2, \ldots, n$. Here the X_i's are *fixed constants* and the ϵ's are i.i.d $N(0, \sigma^2)$ random variables.

(a) Are the Y_i's independent and identically distributed in this case? Explain.

(b) Find the M.L.E's of (β, σ) (7 points).

(3) Let X_1 and X_2 be two i.i.d. observations from an Exponential(λ) distribution. Suppose $X_1 = 1.8$ and $X_2 = 2.6$. Find a confidence interval for λ based on the above data. (6 points)

Note: Tables of quantiles of standard distributions are available in the Appendix Section of Rice's book.