Statistics 406 Last Lab
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1. Generate a timecourse X_t for $t = 1 \dots T$. Let each X_t be iid standard normal. And examine a plot of the data.

```
T<-300
X <- array(rnorm(T), T)
plot(X,t='l')
```

2. Generate a new timecourse $Y_t = X_t + \cos(2\pi t/50)$ using the X_t you generated in the last problem. Make a plot of the cosine term, then make a plot of the cosine term plus the noise.

```
C <- cos(2*pi*(1:T)/50)
Y <- X+C
plot(C)
plot(Y)
```

3. Smooth and standardize Y_t using lowess smoothing with parameter $f = 0.1$. Now look at a plot of the data.

```
S <- lowess(Y,f=0.1)$y
S <- (S-mean(S))/sd(S)
plot(S)
```

4. Consider looking at the data in the frequency domain. To transition to the frequency domain, we will compute the Discrete Fourier Transform (DFT) of our timecourses. Start with $\cos(2\pi t/50)$ and plot the magnitude of the resulting DFT coefficients. Do this for Y_t and the smoothed standardized version of Y_t.

```
## For the cosine term alone
DFTmagnitude <- abs(fft(C))
plot(DFTmagnitude[1:150],t='l')
which.max(DFTmagnitude[1:150])

## For the noise plus the cosine term
DFTmagnitude <- abs(fft(Y))
plot(DFTmagnitude[1:150],t='l')
which.max(DFTmagnitude[1:150])
```

```
## For the smoothed noise and cosine
DFTmagnitude <- abs(fft(S))
plot(DFTmagnitude[1:150],t='l')
which.max(DFTmagnitude[1:150])
```

5. Now construct $Z_t = X_t + \cos(2\pi t/50) + 1_{\{100 \le t \le 110\}}$, where $1_{\{100 \le t \le 110\}} = 1$ when $100 \le t \le 110$ and is zero otherwise. Look at plots and smooth.

```
Z <- Y
Z[100:110] <- Z[100:110]+1
plot(Z)

S <- lowess(Z,f=0.1)$y
plot(S)
```

6. Generate $n = 20$ timecourses of length $T = 300$, where all data starts iid, $X_{it} \sim N(0,1)$ for $i = 1 \ldots 20$ and $t = 1 \ldots 300$. Then add $\delta = 1$ to times 100:110 for half of the timecourses generated.

```
n <- 20
T <- 300
X <- array(rnorm(n*T),c(n,T))
X[1:10, 100:110] <- X[1:10, 100:110] + 1
```

7. For those interested in the Discrete Fourier Transform, here's how its calculated: Given we wish to transform a real valued time sequence $x[n]$ for $n = 0 \ldots N-1$

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-i\frac{2\pi kn}{N}} \qquad \text{for} \quad k = 0 \ldots N-1$$

If we wish to go back to the time domain we can use the inverse Discrete Fourier Transform:

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{i\frac{2\pi kn}{N}} \qquad \text{for} \quad n = 0 \ldots N-1$$