Statistics 406 Lab 4 Notes

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Exercises

1. Let X_i be a sequence of iid Geometric trials.

$$P(X_i = x) = p(1-p)^{x-1} \quad \text{for} x = 1, 2, \ldots$$

and let

$$A_k = (X_1 + \cdots + X_k)/k$$

be their partial averages. Use simulation to show that for k large,

$$Z_k = \sqrt{k}(A_k - EA_k)/\text{sd}(X_i)$$

has a standard normal distribution.

Solution:

```
k <- 1
p <- 0.5
reps <- 1000

notDone <- TRUE

while ( notDone )
{
 #Generate k*reps draws from the modified geometric distribution
 draws <- rgeom(k*reps, p)+1

 X <-array(draws, c(k,reps))
 Ak <- apply( X, 2, mean)

 # Expected value of our geometric sample mean is 1/p
 EAk <- 1/p

 # Std Dev of our geometric RVs Xi is sqrt( (1-p)/p^2 )
 sdk <- sqrt((1-p)/(p^2) )

 Z <- sqrt(k) *(Ak-EAk)/sdk
 diff80 <- abs(quantile(Z, 0.8) - 0.85)
 diff90 <- abs(quantile(Z, 0.9) - 1.28)
```

```
 diff95 <- abs(quantile(Z, 0.95) - 1.65)

 if( diff80 < .05 && diff90 < .05 && diff95 < .05 )
 {
   notDone <- FALSE
 }else {k <- k+1}
}
print(k)
plot(density(Z), main="Distribution of Xbar", ylab = "p(x)", xlab="x")
```

2. Consider a gambler that makes a sequence of bets. She stops when she wins k consecutive bets. Assume the bets are independent, find her expected winnings if for each bet, she wins 1 unit with probability p, and loses one unit with probability $1 - p$.