Statistics 531/Econ 677
Winter, 2005
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SECTION A. We investigate some data from neurophysiology. An electrode implanted (painlessly) into a monkey's brain records a sequence of firing events for an individual neuron cell (neurons communicate by "firing" pulses of electrical charge). Suppose the firing times are $F_1, F_2, \ldots, F_{T+1}$, measured in milliseconds (1ms is 10^{-3}s). We take as our time series $x_t = F_{t+1} - F_t$ with $t = 1, \ldots, T$. This is the series of times intervals between firing events. The data, with $T = 415$, are plotted in Fig. 1. We wish to model x_t in order to quantify the behavior of the neuron, to later compare it with other neurons and investigate the effects of experimental treatments. The estimated ACF and PACF of x_t are shown in Fig. 2.

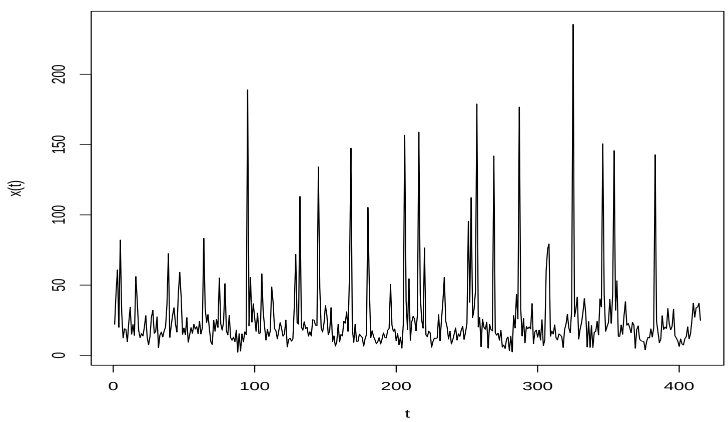

Figure 1: Time series x_t of intervals between neuron firing events

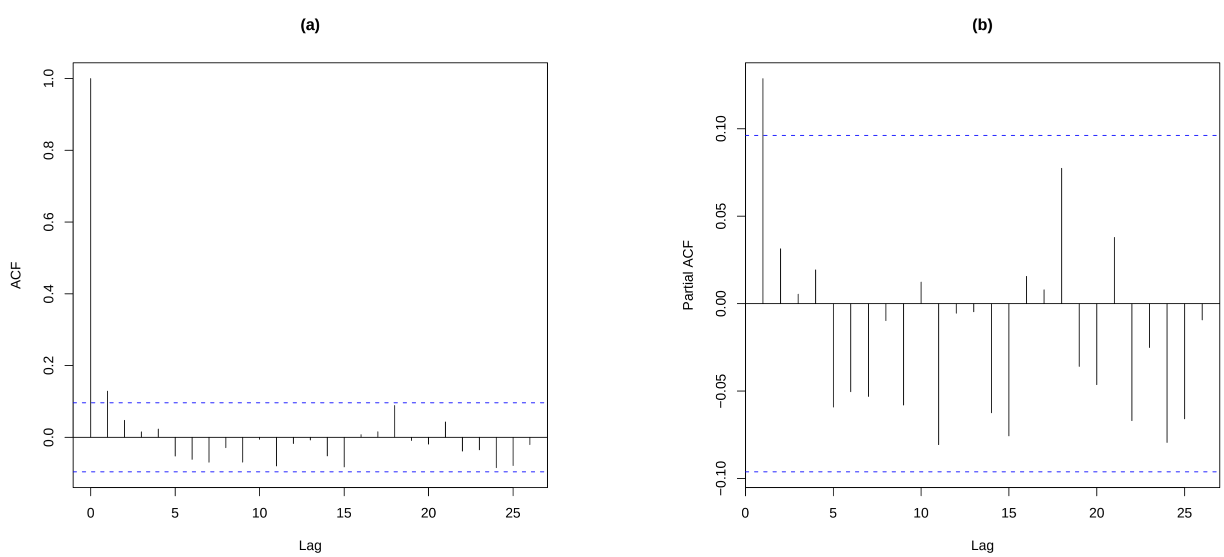

Figure 2: (a) Estimated autocorrelation function of x_t. (b) Estimated partial autocorrelation function of x_t.

A1. [2 pts] What ARMA model for x_t does Fig. 2 suggest, and why?
A2. [2 pts] Another way to select a model is by comparing AIC values. A table of AIC values is shown in Table 1. What ARMA model does this suggest, and why?

$p \setminus q$	0	1	2	3
0	NA	3961.5	3962.7	3964.7
1	3961.0	3962.6	3964.6	3966.6
2	3962.6	3960.5	3959.7	3961.7
3	3964.6	3965.4	3962.6	3968.3

Table 1: AIC values from fitting ARMA(p,q) models to x_t

A3. [2 pts] Find the log likelihood of an ARMA(2,1) model, and explain your calculation.
A4. [2 pts] Does the table of AIC values contain any evidence for or against the claim that the likelihood is correctly calculated and maximized? Explain.
SECTION B. Fitting an ARMA(2,2) model gives the following *R* printout.

```
Call:
arima(x = x, order = c(2, 0, 2))

Coefficients:
         ar1      ar2      ma1     ma2  intercept
      1.6009  -0.6445  -1.4982  0.5219    26.4163
s.e.  0.1886   0.1839   0.2104  0.2094     0.7954

sigma^2 estimated as 791.7:  log likelihood = -1973.88,  aic = 3959.76
```

B1. [4 pts]. Write out the fitted model, carefully stating all the model assumptions.

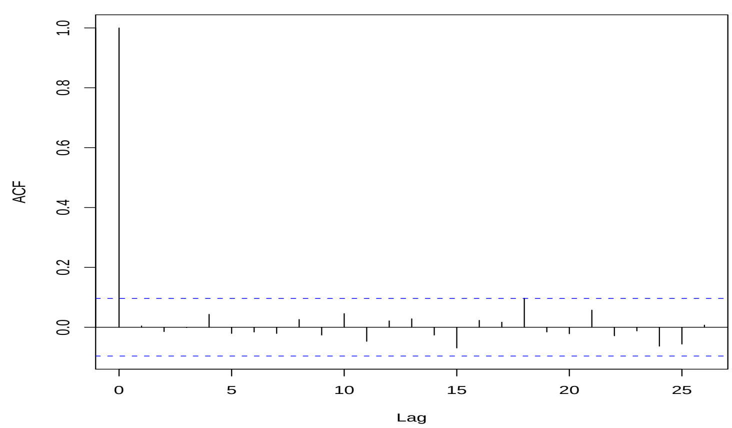

Figure 3: Estimated autocorrelation function of the residuals from fitting an ARMA(2,2) model to x_t.

B2. [3 pts] Fig. 3 shows the ACF of the residuals from fitting an ARMA(2,2) model. Comment on which modeling assumptions this figure supports, and which it does not support.
B3. [3 pts] Find the roots of the AR and the MA polynomials for the fitted ARMA(2,2) model. Is there evidence for parameter redundancy?
B4. [2 pts] Simulations from the fitted ARMA(2,2) model were computed as follows:

```
arma22<-arima(x,order=c(2,0,2))
Nt<-length(x)
sim<-rep(0,Nt)
w<-rnorm(Nt,m=0,sd=sqrt(arma22$sigma2))
for(nt in 3:Nt){
 sim[nt]<-arma22$coef["ar1"]*sim[nt-1]+arma22$coef["ar2"]*sim[nt-2]+
         arma22$coef["ma1"]*w[nt-1]+arma22$coef["ma2"]*w[nt-2]+w[nt]
}
sim<-sim+arma22$coef["intercept"]
```

Sample output is shown in Fig. 4. What does a comparison of Fig. 4 with Fig. 1 say about ARMA modeling of x_t?

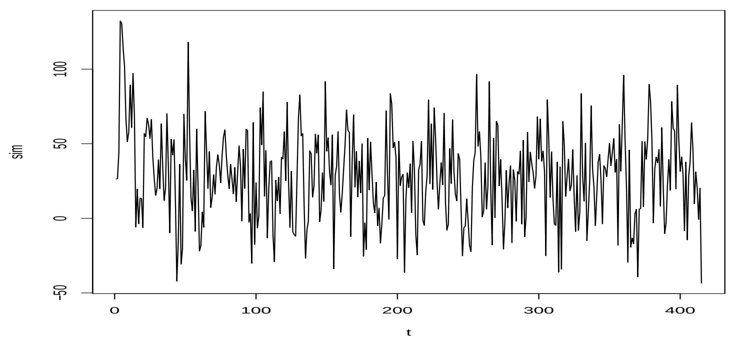

Figure 4: A simulation from the fitted ARMA(2,2) model

B5. [2 pts] Is the random process generated in B4 and plotted in Fig. 4 stationary? Answer yes or no, and explain.

SECTION C. We now investigate a logarithmic transformation of the data. Below is the R printout from fitting an ARMA(2,2) model to $\log_{10} x_t$.

```
Call:
arima(x = log10(x), order = c(2, 0, 2))

Coefficients:
         ar1      ar2      ma1     ma2  intercept
      1.6975  -0.7250  -1.4647  0.4647     1.2925
s.e.  0.0740   0.0718   0.0941  0.0939     0.0021

sigma^2 estimated as 0.07637:  log likelihood = -56.79,  aic = 125.59
```

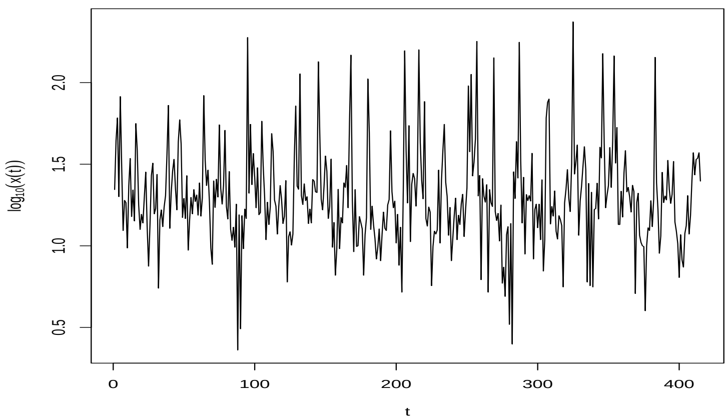

Figure 5: Time plot of $\log_{10} x_t$.

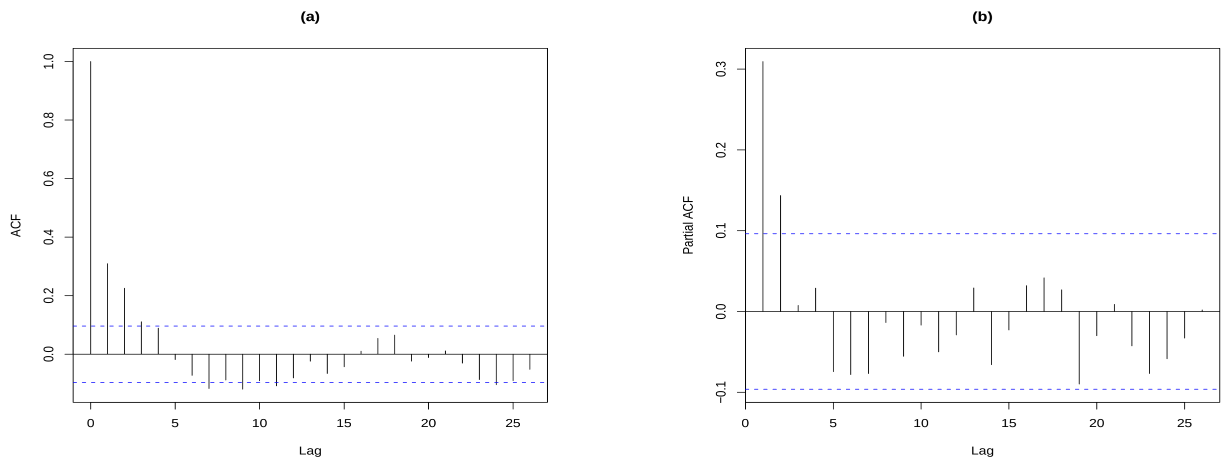

Figure 6: (a) Estimated autocorrelation function of $\log_{10} x_t$. (b) Estimated partial autocorrelation function of $\log_{10} x_t$.

C1. [2 pts] Is there any indication from Fig. 5, Fig. 6 and the fitted model printouts in Sections B and C that ARMA modeling is more successful after a log transformation? or less? Explain.

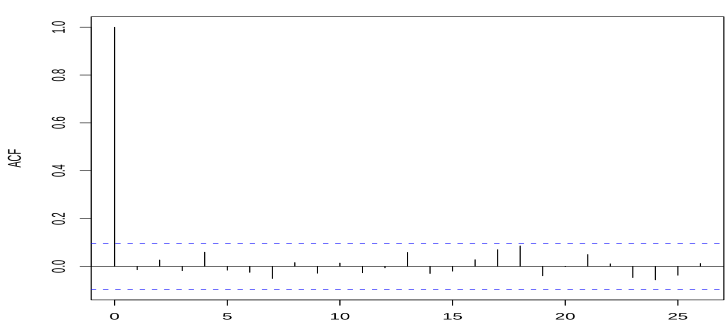

Figure 7: Estimated autocorrelation function of the residuals from fitting an ARMA(2,2) model to $\log_{10} x_t$.

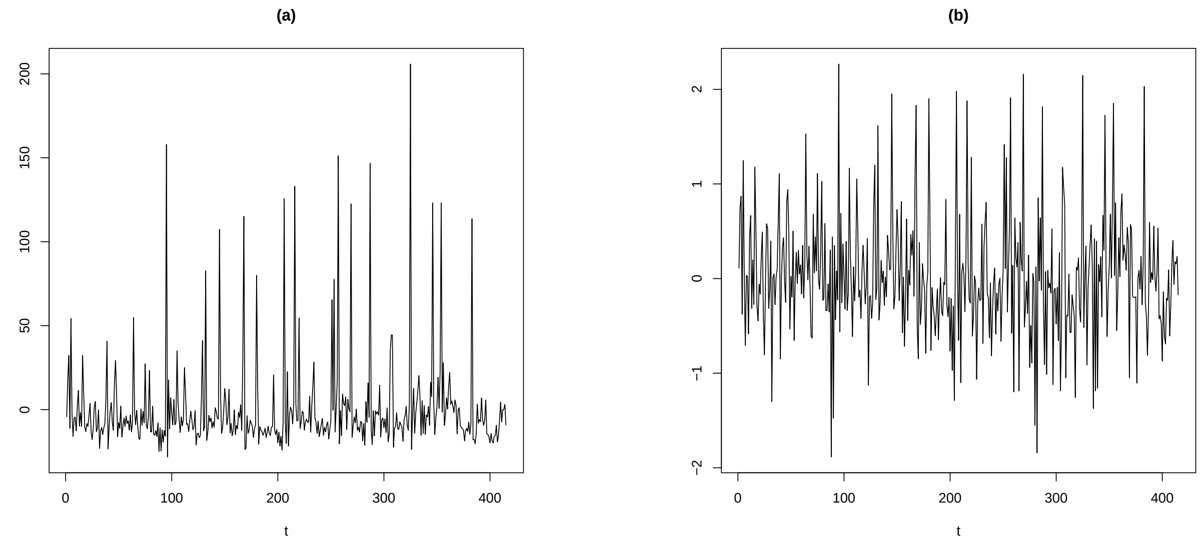

Figure 8: (a) Residuals from fitting an ARMA(2,2) model to x_t. (b) Residuals from fitting an ARMA(2,2) model to $\log_{10} x_t$.

C2. [2 pts] What do Figs. 3, 7 and 8 indicate about the success of the log transform?

C3. [3 pts] Fig. 9 shows the smoothed periodogram of $\log_{10} x_t$. Find the frequency and period corresponding to the peak in the periodogram. Your answer should include the units of these quantities. Describe briefly what this peak leads you to conclude about how the monkey's neuron behaves.

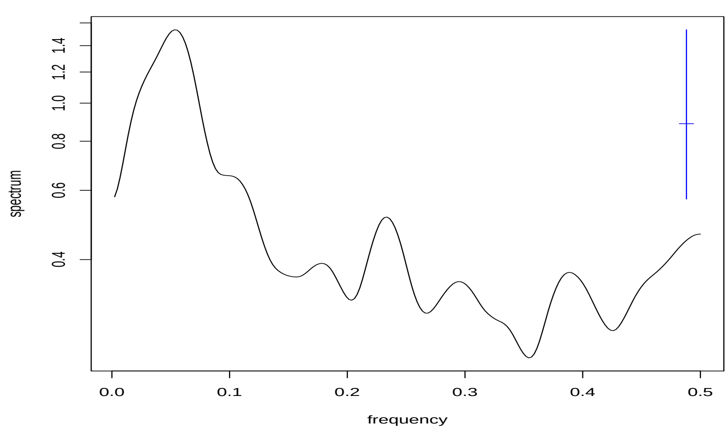

Figure 9: Smoothed periodogram of $\log_{10} x_t$.