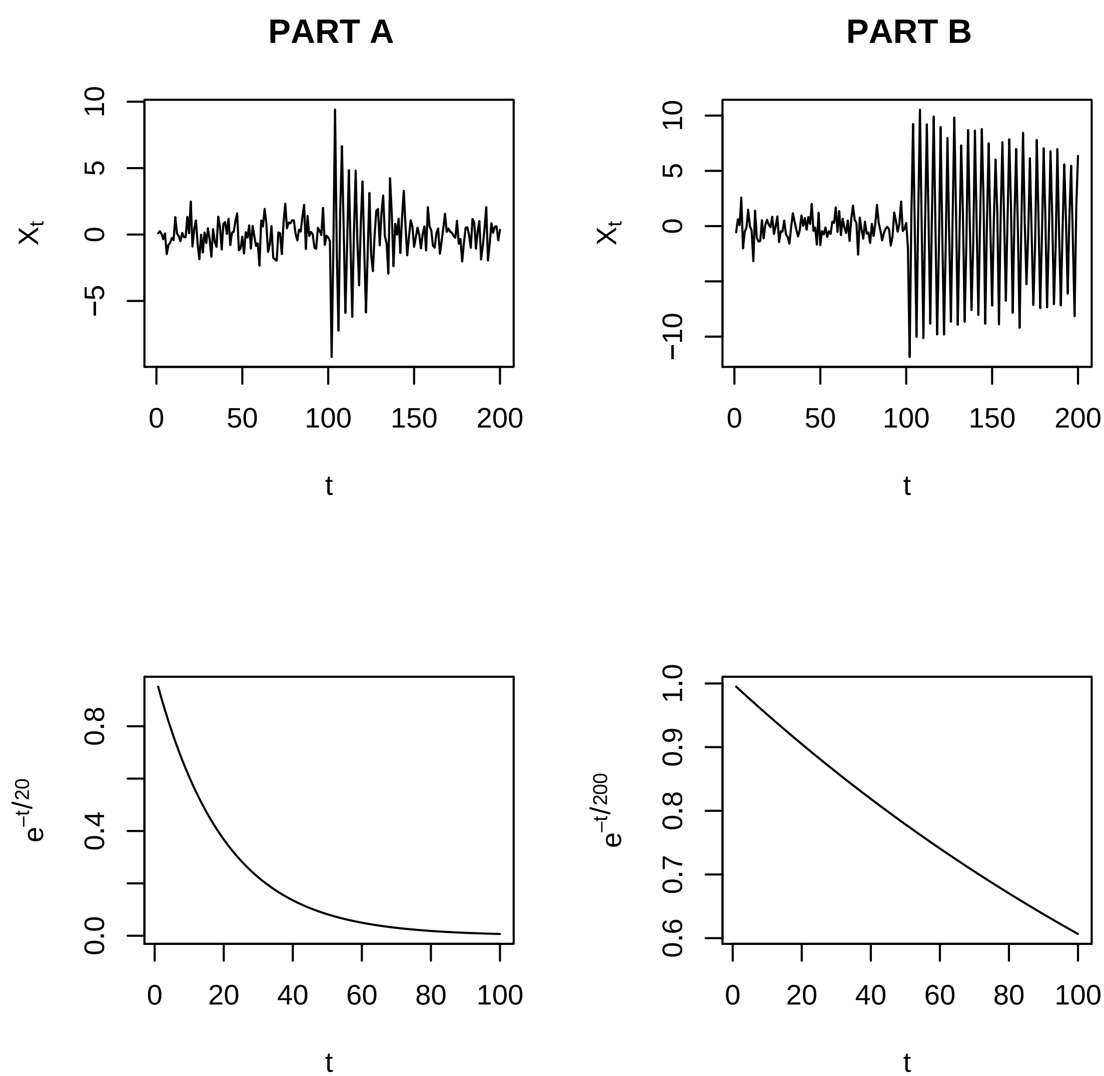
## STATS 531 / ECON 677 WINTER 09
## HOMEWORK 1

### PROBLEM 1.2

**(A),(B)**

**(C)** Series (a) shares more features with the explosion series while series (b) looks more like the earthquake series. The modulator of series (b) declines almost linearly while the modulator of series (a) declines much faster.




### PROBLEM 1.6

**(A)** Note that $E(x_t) = \beta_1 + \beta_2 t$ and $\gamma_x(h) = \gamma_w(h)$. It follows that $x_t$ is not stationary for $\beta_2 \neq 0$, because the mean depends on $t$, and is weakly stationary for $\beta_2 = 0$, since the mean does not depend on $t$ and the covariance function depends on $s$ and $t$ only through their difference.

**(B)** Note that $y_t = x_t - x_{t-1} = \beta_2 + w_t - w_{t-1}$. It follows that $E(y_t) = \beta_2$ and

$$\begin{aligned} \gamma_y(h) &= E[(w_{t+h} - w_{t+h-1})(w_t - w_{t-1})] \\ &= \begin{cases} 2\sigma_w^2 & \text{if } h = 0 \\ -\sigma_w^2 & \text{if } |h| = 1 \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

Hence, $y_t$ is weakly stationary because its mean does not depend on $t$ and the covariance function depends on $s$ and $t$ only through their difference.

**(C)** Note that

$$\begin{aligned} v_t &= \frac{1}{2q+1} \sum_{j=-q}^{q} [\beta_1 + \beta_2(t-j) + w_{t-j}] \\ &= \beta_1 + \beta_2 t - \frac{1}{2q+1}(\beta_2 \sum_{j=-q}^{q} j - \sum_{j=-q}^{q} w_{t-j}) \\ &= \beta_1 + \beta_2 t + \frac{1}{2q+1} \sum_{j=-q}^{q} w_{t-j} \end{aligned}$$

since the $j$s in the sum cancel out. It follows that $E(v_t) = \beta_1 + \beta_2 t$. To obtain a simplified expression for the covariance function,

$$\begin{aligned} \gamma_v(h) &= E[(v_{t+h} - E(v_{t+h}))(v_t - E(v_t))] \\ &= E[(\frac{1}{2q+1} \sum_{j=-q}^{q} w_{t+h-j})(\frac{1}{2q+1} \sum_{j=-q}^{q} w_{t-j})] \\ &= \frac{1}{(2q+1)^2} E[\sum_{j=-q}^{q} w_{t+h-j} \sum_{j=-q}^{q} w_{t-j}] \\ &= \frac{1}{(2q+1)^2} E[\sum_{j=-q}^{q} \sum_{k=-q}^{q} w_{t+h-j} w_{t-k}] \\ &= \frac{1}{(2q+1)^2} \sum_{j=-q}^{q} \sum_{k=-q}^{q} \gamma_w(t+h-j, t-k) \end{aligned}$$

Hence,

$$\gamma_v(h) = \begin{cases} \frac{\sigma_w^2(2q+1-|h|)}{(2q+1)^2} & \text{if } |h| \leq 2q+1 \\ 0 & \text{otherwise} \end{cases}$$