STATS 531 / ECON 677 WINTER 09
HOMEWORK 6

PROBLEM 4.23

(A) Using property P.4.4 in the book (page 221), it follows that the spectrum of the filtered outputs are

$$\begin{aligned} f_{yy}(\omega) &= |A_{yx}(\omega)|^2 f_{xx}(\omega) \\ f_{zz}(\omega) &= |B_{zy}(\omega)|^2 f_{yy}(\omega). \end{aligned}$$

The result follows by plugging f_{yy} from the first equality in the second one,

$$f_{zz}(\omega) = |B_{zy}(\omega)|^2 |A_{yx}(\omega)|^2 f_{xx}(\omega).$$

(B) Taking the first difference of the input signal (i.e., applying the high-pass filter u_t) will enhance high frequencies by attenuating low frequencies (in particular, it removes a linear trend). Taking a difference of order 12 (i.e. applying the filter v_t) will remove seasonal frequencies corresponding to cycles that complete every 12 time periods (as well as frequencies corresponding to multiples of 12). Using the two filters will combine the two effects.

(C) Let $\theta = 2\pi\omega$. By definition, $A_{yx}(\omega) = 1 - e^{-i\theta} = 1 - \big(\cos(-\theta) + i\sin(-\theta)\big)$. An expression for $|A_{yx}(\omega)|^2$ follows from

$$\begin{aligned} |(1-e^{-i\theta})|^2 &= Re(1-e^{-i\theta})^2 + Im(1-e^{-i\theta})^2 \\ &= \big(1-\cos(-\theta)\big)^2 + \big(-\sin(-\theta)\big)^2 \\ &= 1 - 2\cos(-\theta) + \cos^2(-\theta) + \sin(-\theta) \\ &= 1 - 2\cos(-\theta) + 1 = 2 - 2\cos(\theta) = 2\big(1-\cos(2\pi\omega)\big), \end{aligned}$$

where the first equality follows by definition and the fourth by trigonometric identities. Similarly $B_{zy}(\omega) = 1 - e^{-12i\theta} = 1 - \big(\cos(-12\theta) + i\sin(-12\theta)\big)$ and $|B_{zy}(\omega)|^2 = 2\big(1 - \cos(24\pi\omega)\big)$. The functions are plotted in figure 1.

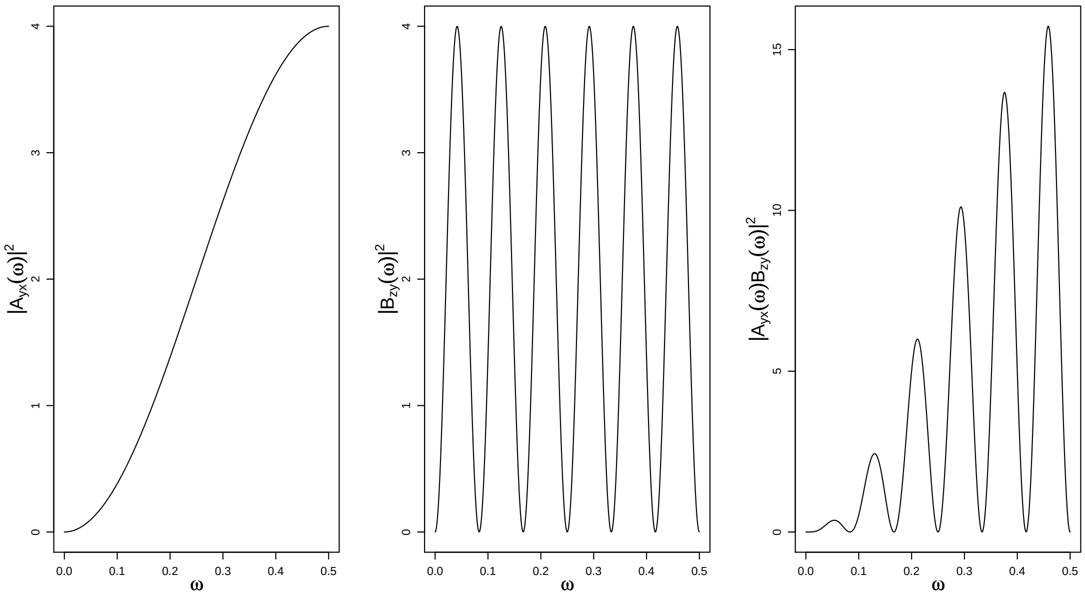

Figure 1: Square of the absolute value of frequency responses of the filters

PROBLEM 5.9 The regression model can be fit using R in 2 ways: using the stepwise procedure described in the book or using the `xreg` argument in the `arima` function. We use the second option to obtain the following AIC table

```
      MA(0) MA(1) MA(2) MA(3)
AR(0) 418.3 361.3 354.8 352.8
AR(1) 345.4 347.3 344.8 346.7
AR(2) 347.4 347.8 346.7 348.2
AR(3) 344.5 346.3 345.1 347.1
```

For orders larger than 3, R issues convergence warnings so we do not explore those. Note as well that the AIC decreases by more than 2 going from ARMA(2,0) to ARMA(3,0) and by almost 2 going from ARMA(3,1) to ARMA(3,0). This suggests that there could be some numerical issue so we choose the ARMA(1,0) model (instead of ARMA(3,0) which minimizes the AIC in the table). The output of the fit is

```
> arima(dsales,order=c(1,0,0),xreg=dlead)

Call: arima(x = dsales, order = c(1, 0, 0), xreg = dlead)

Coefficients:
         ar1  intercept   dlead
```

```
      0.6451     0.3624  2.7876
s.e.  0.0628     0.1767  0.1432

sigma^2 estimated as 0.5884:  log likelihood = -168.72,  aic=345.43.
>
```

Thus the fitted model is

$$\nabla S_t = 0.36 + 2.79 \nabla L_{t-3} + x_t,$$

where $x_t = 0.64 x_{t-1} + w_t$ with $\sigma_w^2 = 0.59$. Inspection of the diagnostics in figure 2 does not suggest that any of the assumptions of the model is violated.

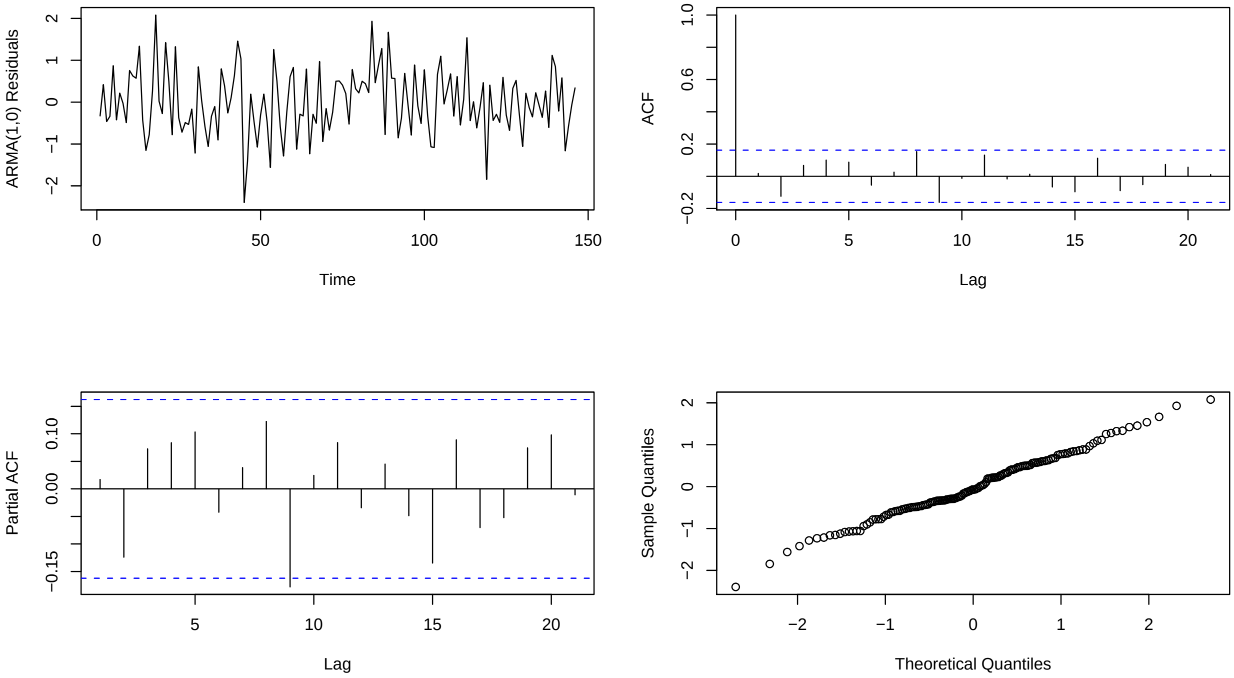

Figure 2: Residual plots