STATS 531 / ECON 677 WINTER 09
HOMEWORK 8

PROBLEM 4.34

(A) Let I_t be the transformed inflow and P_t the transformed precipitation as in the textbook. Figure 1 shows the smoothed periodogram of I_t and the square coherence of each weather series and I_t. The hump in the power spectrum between $\omega = 0.041$ and $\omega = 0.125$ (between 24 and 8 months), peaking at $\omega = 0.083 = 1/12$ indicates strong annual cycles (a feature shared by all series except wind, as can be seen by the high square coherence around that frequency). Apart from this hump, the power spectrum of I_t concentrates between $\omega = 0.125$ and $\omega = 0.32$ (between 8 and 3 months). At this frequencies (and also at the lower ones), I_t and P_t appear to be consistently more coherent than I_t and any of the other series. This means that the squared correlation between I_t and P_t is larger at the most relevant frequencies than between I_t and the other variables, which makes P_t the strongest determinant of I_t.

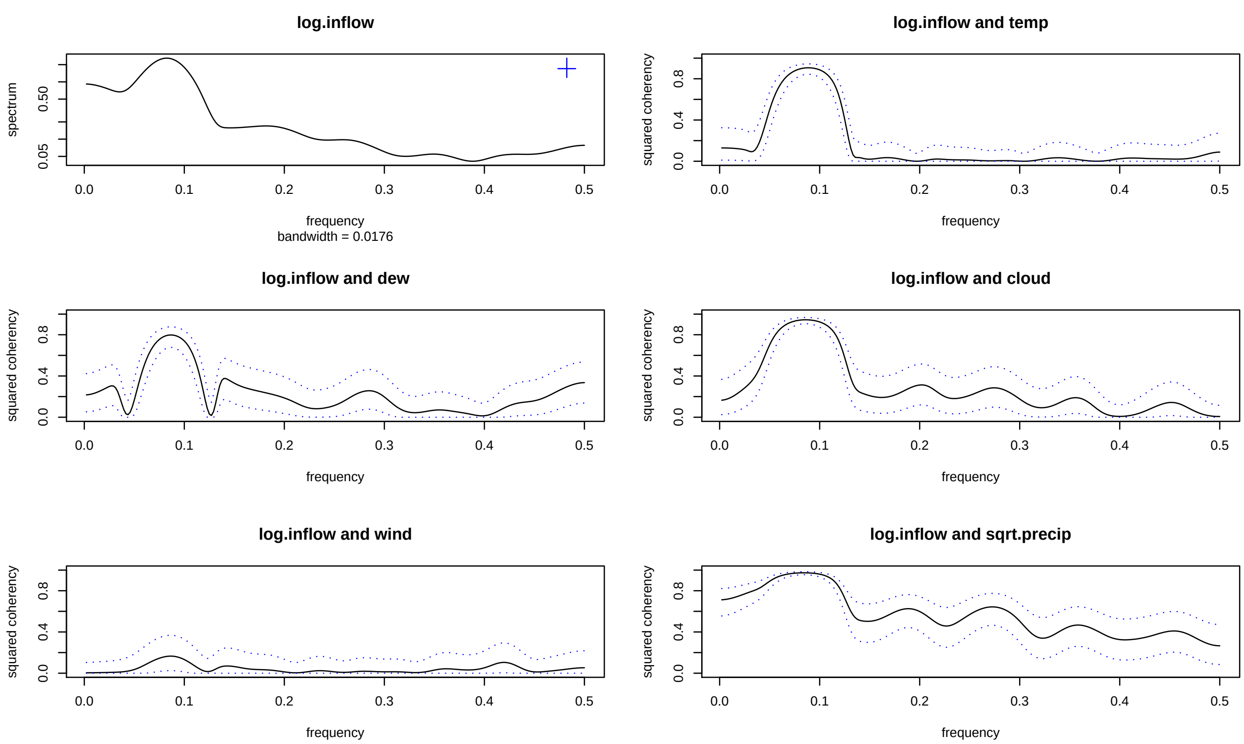

Figure 1: Partial cross-spectrum of the series

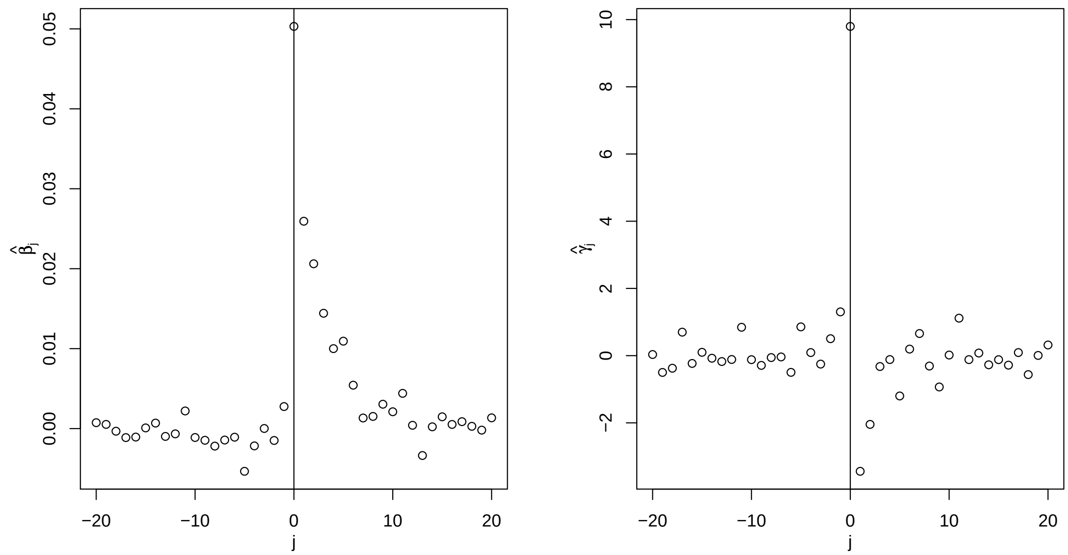

Figure 2: Estimated impulse response functions

(B) Figure 2 shows estimates of the impulse response functions $\{\beta_s\}_{t\in\mathbb{Z}}$ and $\{\gamma_s\}_{t\in\mathbb{Z}}$ in the convolutions

$$\begin{aligned} I_t &= \sum_{s=-\infty}^{\infty} \beta_s P_{t-s} \\ P_t &= \sum_{s=-\infty}^{\infty} \gamma_s I_{t-s} \end{aligned}$$

using the cross-spectrum (and assuming stationary processes with zero mean). The fact that the estimates $\hat{\gamma}_0$ and $\hat{\gamma}_1$ appear to be large in magnitude compared to the rest supports a model similar to the one suggested in the problem, i.e.

$$P_t = 9.80 I_t - 3.36 I_{t-1} + \omega_t.$$

Another reasonable model would include a term like $\beta_2 I_{t-2}$, since $\hat{\beta}_2$ is also relatively large. And other models could be based on the estimates $\hat{\beta}_s$. Note that we do not use standard errors. Further analysis would include fitting different models, testing significance of coefficients and model selection using the AIC. Going back to suggested model above, dividing through by 9.80 and rearranging terms in the one gets

$$\begin{aligned} \frac{P_t}{9.80} &= I_t - \frac{3.36}{9.80} I_{t-1} + \frac{\omega_t}{9.80} \\ 0.1P_t &= (1 - 0.343B) I_t + 0.1\omega_t \\ I_t &= \frac{0.1(\omega_t - P_t)}{1 - 0.343B}, \end{aligned}$$

which suggests that 0.343 is a reasonable value for ϕ.