Statistics 620
Fall, 2006

Name: ______________________________ **UMID #:** ______________________________

Midterm Exam

- **There are 3 questions, each worth 10 points.**
- **You are allowed a calculator and a single-sided sheet of notes.**
- **Credit will be given for clear explanation and justification, as well as for getting the correct answer.**
- **Cross out any working that you do not wish to be considered as part of your solution. You are advised not to erase unfinished working since partial credit may be available for an indication that an appropriate method was attempted, even if it was later rejected.**

Problem	Points	Your Score
1	10	
2	10	
3	10	
Total	30	

1. Workplace accidents occur at University of Michigan as a Poisson process with rate λ. Each accident independently results in a lawsuit with probability p. Let T denote the time of the first lawsuit, and let A be the total number of accidents in the random time interval $[0, T]$. Find $P[A = n | T = t]$.

   Hint: you may (if you wish) use without proof the result that, if $N(t)$ is a Poisson process and each event is independently classified as Type I or Type II, then the corresponding counting processes $N_1(t)$ and $N_2(t)$ for events of each type are independent Poisson processes.

2. Let $\{N(t), t \geq 0\}$ be a renewal process, with corresponding arrival times $\{S_n\}$ and inter-arrival times given by $X_n = S_n - S_{n-1}$. Suppose X_n has distribution F. Define the age process by $A(t) = t - S_{N(t)}$, namely the time since the most recent arrival.

(i) Show that $E[A(t)] = t\bar{F}(t) + \int_0^t (t-u)\bar{F}(t-u)dm(u)$.

Hint: recall that $dF_{S_{N(t)}}(u) = \begin{cases} \bar{F}(t) + \bar{F}(t)dm(0) & \text{for } u = 0 \\ \bar{F}(t-u)dm(u) & \text{for } u > 0 \end{cases}$

(ii) Hence, show that $\lim_{t\to\infty} E[A(t)] = E[X^2]/2E[X]$, where X has distribution F.

3. Trials are performed in sequence. If the two most recent previous trials were both successes, the next trial is a success with probability 0.8; otherwise, the chance of success is 0.5.

   (i) Define state 1 to be {most recent trial was a failure}, state 2 to be {most recent trial was a success, and the preceding trial was a failure} and state 3 to be {last two trials were successes}. Let X_n be the state after the nth trial. Explain why $\{X_n\}$ is a Markov chain, and find the transition matrix $P = [P_{ij}]$.

   (ii) Use the Markov chain in (i) to find the long run proportion of trials that are successes.