Statistics 620
Fall, 2008

Name: ______________________________ **UMID #:** ______________________________

Midterm Exam

- **There are 4 questions, each worth 10 points.**
- **You are allowed a calculator and a single-sided sheet of notes.**
- **Credit will be given for clear explanation and justification, as well as for getting the correct answer.**
- **Cross out any working that you do not wish to be considered as part of your solution. You are advised not to erase unfinished working since partial credit may be available for an indication that an appropriate method was attempted, even if it was later rejected.**

Problem	Points	Your Score
1	10	
2	10	
3	10	
4	10	
Total	40	

1. A certain protein has two stable conformations, labeled 1 and 2 (a conformation is a configuration, or folding, of the protein molecule; here we need to know only that the protein can be in one of two distinct states). The protein is observed at a sequence of times $0, 1, 2, \ldots$. At time 0, the protein is in conformation 1. From conformation 1, there is a chance of 0.1 of having moved into conformation 2 for the next observation time. From conformation 2, the chance of moving to conformation 1 is 0.2. Find the chance that the protein is in conformation 1 at time n (for $n = 0, 1, 2, \ldots$). You may, if you wish, write your answer in terms of the solutions λ_1 and λ_2 to the characteristic equation $\begin{vmatrix} 0.9-\lambda & 0.1 \\ 0.2 & 0.8-\lambda \end{vmatrix} = 0$ for the matrix $P = \begin{pmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{pmatrix}$.

2. Let $N(t)$ be a renewal process having interarrival times $X_1, X_2, \ldots$ with distribution F. By constructing an alternating renewal process, derive an expression for the limiting probability (as $t \to \infty$) that the renewal interval containing time t has length $\leq x$. Evaluate this expression in the special case when $N(t)$ is a Poisson process with rate λ. Show that, in this case, the distribution of the length of the interval containing t is different from F (in fact, you may also notice that these distributions differ in general, as well as for the particular case of a Poisson process).

The next two questions relate to two definitions A and B below for a counting process to be a Poisson process with rate λ:

A. (i) $N(0) = 0$.
(ii) $N(t)$ has independent increments.
(iii) The number of events in any interval of length t has a Poisson distribution with mean λt.

B. (i) $N(0) = 0$.
(ii) $N(t)$ has stationary independent increments.
(iii) $P\{N(h){=}1\} = \lambda h + o(h)$.
(iv) $P\{N(h){\geq}2\} = o(h)$.

3. Show that A implies B.

4. Show that B implies A.

Hint: one way to do this involves dividing up an interval $[0, t]$ into n equal pieces and showing that $N(t)$ is well approximated by a binomial random variable related to the increments of $N(t)$ on these n pieces; then take a limit as $n \to \infty$.