Solution to Assignment 10

9.24: **(a)** Let likelihood function is $L(p) = \binom{n}{x} p^X (1-p)^{n-X}$, maximized when $p = \hat{p} \stackrel{\text{def}}{=} X/n$. So

$$\Lambda = \frac{L(1/2)}{L(\hat{p})} = \frac{(n/2)^n}{X^X(n-X)^{n-X}}.$$

(b) Let $\Delta = X - n/2$. Then

$$\Lambda = \frac{(n/2)^n}{\left(\frac{n}{2}+\Delta\right)^{n/2+\Delta}\left(\frac{n}{2}-\Delta\right)^{n/2-\Delta}} = \frac{(n/2)^n}{\left(\frac{n}{2}+|\Delta|\right)^{n/2+|\Delta|}\left(\frac{n}{2}-|\Delta|\right)^{n/2-|\Delta|}}.$$

So $\Lambda = g(|X-n/2|)$ with

$$\log[g(u)] = n\log(n/2) - (n/2+u)\log(n/2+u) - (n/2-u)\log(n/2-u).$$

But for $u > 0$,

$$\frac{d}{du}\log[g(u)] = \log\left(\frac{n/2-u}{n/2+u}\right) < 0.$$

This shows that g is a strictly decreasing function on $(0,\infty)$. So Λ is a strictly decreasing function of $|X-n/2|$, and the generalized likelihood ratio test will reject H_0 if $|X-n/2|$ is large enough. **(c)** Summing the mass function,

$$\alpha = \sum_{x:|x-n/2|>k} \binom{n}{x}(1/2)^x(1/2)^{n-x} = \frac{2}{2^n}\sum_{x<n/2-k}\binom{n}{x}.$$

(d) $\alpha = 10.9\%$. **(e)**

$$\alpha = 1 - P_{H_0}(40 \le X \le 60) = 1 - P_{H_0}\left(-2 \le \frac{X-50}{5} \le 2\right)$$
$$\approx 1 - \Phi(2) + \Phi(-2) = 4.56\%.$$

(With a continuity correction, the approximation for α would be 3.58%.)

9.26: **(a)** True. **(b)** False. **(c)** True. **(d)** False. **(e)** False. **(f)** False.

9.40: Since $X_1 + X_2 = n$ and $p_1 + p_2 = 1$, $X_2 - np_2 = np_1 - X_1$. Using this, Pearson's chi-square statistic is

$$\frac{(X_1-np_1)^2}{np_1} + \frac{(X_1-np_1)^2}{n(1-p_1)} = \frac{(X_1-np_1)^2}{np_1(1-p_1)}.$$