Solution to Assignment 2

2.40: **(a)** $c = 3$. **(b)**

$$F_X(x) = \begin{cases} 0, & x \le 0; \\ x^3, & 0 < x < 1; \\ 1, & x \ge 1. \end{cases}$$

(c) $P(.1 \le X \le .5) = 0.124$.

2.60: The density is

$$\frac{1}{y\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(\log y - \mu)^2}{2\sigma^2}\right], \qquad y > 0.$$

3.6: The marginal densities are

$$f_X(x) = \frac{2\sqrt{a^2 - x^2}}{\pi a^2}, \qquad -a < x < a,$$

$$f_Y(y) = \frac{2\sqrt{b^2 - y^2}}{\pi b^2}, \qquad -b < y < b.$$

3.72: $P(X_{(n)} \le y) = P(X_{(n)} \le y, X_{(1)} \le x) + P(X_{(n)} \le y, X_{(1)} > x)$. But

$$P(X_{(n)} \le y) = P(X_i \le y, i = 1, \ldots, n) = F^n(y)$$

and

$$P(X_{(n)} \le y, X_{(1)} > x) = P(x < X_i \le y, i = 1, \ldots, n) = \left[F(y) - F(x)\right]^n.$$

4.44: $\mathrm{Cov}(X + Y, X - Y) = 0$.

4.54: $\mathrm{Cov}(U, V) = \sigma_Z^2$, and

$$\rho_{U,V} = \frac{\sigma_Z^2}{\sqrt{(\sigma_Z^2 + \sigma_X^2)(\sigma_Z^2 + \sigma_Y^2)}}.$$