Solution to Assignment 3

4.80: The momemt function and its derivatives are:

$$M(t) = \frac{(2t-2)e^t+2}{t^2}, \qquad M'(t) = \frac{(2t^2-4t+4)e^t-4}{t^2},$$

and

$$M''(t) = \frac{(2t^3-6t^2+12t-12)e^t+12}{t^4}.$$

By L'Hopital's rule, or Taylor expansion, $M'(0) = 2/3$ and $M''(0) = 1/2$. Direct calculation gives the same values: $E[X] = \int_0^1 2x^2\,dx = 2/3$ and $E[X^2] = \int_0^1 2x^3\,dx = 1/2$.

4.88: $M_Z(t) = M_X(\alpha t) \times M_Y(\beta t)$.

5.10: **(a)** $E(X) = 16.67$ and $\text{Var}(X) = 13.89$. So

$$P(15 < X < 20) = P\left(-.45 < \frac{X-16.67}{\sqrt{13.88}} < .89\right)$$
$$\approx \Phi(.89) - \Phi(-.45) = .4869.$$

(b) $E(X) = 350$ and $\text{Var}(S) = 291.67$. So

$$P(S < 300) = P\left(\frac{S-350}{\sqrt{291.67}} < -2.93\right) \approx \Phi(-2.93) = .0017.$$

(The numbers will be slightly different with a continuity correction.)

5.18: $E(T) = 1500$ and $\text{Var}(T) = 10,000$. So

$$P(T > 1700) = P\left(\frac{T-1500}{100} > 2\right) \approx 1 - \Phi(2) = .0228.$$

6.6: If $Z \sim N(0,1)$ and $U \sim \chi^2_n$, then

$$T \sim \frac{Z}{\sqrt{U/n}} \qquad \text{and} \qquad T^2 \sim \frac{Z^2}{U/n},$$

which has the $F_{1,n}$ distribution since $Z^2 \sim \chi^2_1$ and Z^2 and U are independent.