Solution to Assignment 6

8.8: **(a)** $\hat{p} = 1/\overline{X} = 0.358$. **(b)** The estimated standard error of $\hat{p}$ is

$$s_{\hat{p}} = \frac{1}{\sqrt{nI(\hat{p})}} = \sqrt{\frac{(1-\hat{p})\hat{p}^2}{n}} = 0.0252.$$

So an approximate 95% confidence interval is

$$(\hat{p} \pm 1.96 s_{\hat{p}}) = (0.358 \pm 0.049).$$

(c) The expected counts under the fitted model are 46.5, 29.8, 19.2, 12.3, 7.9, 5.1, 3.3, 2.1, 1.3, 0.9, 0.6, and 0.4. These agree resasomably well with the observed frequencies.

8.16: **(c)** The asymptotic variance is $1/\big(nI(\sigma)\big) = \sigma^2/n$.

8.50: **(a)** Since

$$E_\theta(X) = \frac{1}{\theta^2}\int_0^\infty x^2 e^{-x^2/(2\theta^2)}\,dx = \theta\sqrt{\pi/2},$$

$\hat{\theta}_{\mathrm{MOM}} = \overline{X}\sqrt{2/\pi}$. **(b)** The maximum likelihood estimator is

$$\hat{\theta}_{\mathrm{MLE}} = \sqrt{\frac{1}{2n}\sum_{i=1}^n X_i^2}.$$

(c) The asymptotic variance is $1/\big(nI(\theta)\big) = \theta^2/(4n)$. **(d)** The approximate confidence interval is

$$\left(\hat{\theta}_{\mathrm{MLE}} \pm \frac{1.645\hat{\theta}_{\mathrm{MLE}}}{2\sqrt{n}}\right).$$

8.56: $\hat{\theta}_1 = 0.0808$, $\hat{\theta}_4 = 0.0333$. Since $X_1 \sim \mathrm{Binomial}\big(n, (2+\theta)/4\big)$, $E[X_1] = n(2+\theta)/4$ and

$$E[\hat{\theta}_1] = E\left[\frac{4X_1}{n} - 2\right] = \frac{4}{n}E[X_1] - 2 = \theta.$$

Similarly, since $X_4 \sim \mathrm{Binomial}(n, \theta/4)$, $E[X_4] = n\theta/4$ and $E[\hat{\theta}_4] = \theta$. Since $\mathrm{Var}(X_1) = n(4-\theta^2)/16$ and $\mathrm{Var}(X_4) = n\theta(4-\theta)/16$,

$$\mathrm{Var}(\hat{\theta}_1) = \frac{16\mathrm{Var}(X_1)}{n^2} = \frac{4-\theta^2}{n}$$

and

$$\mathrm{Var}(\hat{\theta}_4) = \frac{16\mathrm{Var}(X_4)}{n^2} = \frac{\theta(4-\theta)}{n}.$$

Plugging in estimates for θ, the estimated standard errors are $s_{\hat{\theta}_1} = 0.0323$ and $s_{\hat{\theta}_4} = 0.0061$. The maximum likelihood estimate is $\hat{\theta} = 0.036$ with $s_{\hat{\theta}} = 1/\sqrt{nI(\hat{\theta})} = 0.00586$. So $s_{\hat{\theta}} < s_{\hat{\theta}_4} < s_{\hat{\theta}_1}$.