Solution to Assignment 7

8.4. **(e)** The posterior density is

$$f_{\Theta|X}(\theta|x) = \frac{\Gamma(12)}{\Gamma(6)\Gamma(6)}\theta^5(1-\theta)^5, \qquad 0 \le \theta \le 1,$$

with mode 1/2.

8.8. **(d)** The posterior density is

$$f_{P|X}(p|x) = \frac{\Gamma(365)}{\Gamma(131)\Gamma(234)}p^{130}(1-p)^{233}, \qquad 0 \le p \le 1,$$

which is a Beta density with parameters 130 and 233. The posterior mean and variance are

$$E[P|X=x] = \frac{131}{365} = 0.359$$

and

$$\mathrm{Var}(P|X=x) = \frac{131 \times 233}{365^2 \times 366} = 0.000626.$$

So the posterior standard deviation is $\sqrt{0.000626} = 0.025$.

8.66. **(a)** The posterior density of Θ is

$$f_{\Theta|X}(\theta|x) = 3\theta^2, \qquad 0 \le \theta \le 1,$$

(b) By smoothing,

$$P(S_1S_2) = E[P(S_1S_2|\Theta)] = E[\Theta^2] = \int_0^1 \theta^2\, d\theta = 1/3$$

and

$$P(S_1S_2S_3) = E[P(S_1S_2S_3|\Theta)] = E[\Theta^3] = \int_0^1 \theta^3\, d\theta = 1/4.$$

So

$$P(S_3|S_1S_2) = \frac{P(S_1S_2S_3)}{P(S_1S_2)} = \frac{1/4}{1/3} = \frac{3}{4}.$$

This answer equals $E(\Theta|S_1S_2)$, which can also be justified by a smoothing argument.