PRACTICE EXAM—Statistics 426

The midterm will be closed book and notes, but you may bring a formula sheet (8.5×11) and a calculator.

(1) Let X be a random variable with probability density function

$$f_X(x) = \begin{cases} 2x, & x \in (0,1); \\ 0, & \text{otherwise}, \end{cases}$$

and let $Y = X^2$.

(a) Find the variance of X.
(b) Find the cumulative distribution function for Y. What is the density function for Y?

(2) Let X and Y have joint probability density function

$$f(x,y) = \begin{cases} 3x, & 0 < y < x < 1; \\ 0, & \text{otherwise}, \end{cases}$$

(a) Find the marginal density f_Y of Y.
(b) Find $P(2Y > X)$.
(c) Compute $E[XY]$.

(3) If X and Y are independent, each with the standard exponential density (e^{-x} for $x > 0$), find $P(2X > Y)$.

(4) Let X be a discrete random variable with mass function

$$p(x) = \frac{1}{x!(e-1)}, \qquad x = 1, 2, \ldots.$$

(a) Find the moment generating function for X.
(b) Find $E[X]$.

(5) Let X_1, X_2, and X_3 be independent variables with $\mathrm{Var}(X_1) = \mathrm{Var}(X_2) = \mathrm{Var}(X_3) = \sigma^2$. Determine the covariance between $X_1 + X_2$ and $X_2 + X_3$.

(6) A gambler plays roulette repeatedly, betting \$1 on "red" each time. So on any given play he wins \$1 with probability 9/19 and but loses \$1 with probability 10/19. Find the chance (approximately) the gambler is a net winner after 1,000 plays.

(7) A population has four units with values 1, 2, 4, and 4. Let M denote the median of a simple random sample of size 3 from this population. (The median is the middle value of the three units in the sample.) Find the probability mass function for M.

(8) Let $X_1, \ldots, X_n$ be i.i.d. continuous random variables with common density

$$f(x|\theta) = \begin{cases} \frac{1}{2}(1 + x\theta), & -1 < x < 1; \\ 0, & \text{otherwise,} \end{cases}$$

where $\theta \in [-1, 1]$ is an unknown parameter. Derive a formula for the method moments estimator of θ.

(9) Let $X_1, \ldots, X_n$ be i.i.d. from the uniform distribution on $[-\theta, \theta]$. Find the maximum likelihood estimator of θ.

(10) Let $X_1, \ldots, X_n$ be i.i.d. continuous random variables with common density

$$f(x|\theta) = \begin{cases} x\theta^2 e^{-\theta x}, & x > 0; \\ 0, & \text{otherwise,} \end{cases}$$

where $\theta > 0$ is an unknown parameter.

(a) Find the maximum likelihood estimator of θ.
(b) Find the Fisher information $I(\theta)$ (for a single observation).
(c) Give an approximate 95% confidence interval for θ.