Limit Theorems for Random Walks Conditioned to Stay Positive

Robert W. Keener

Let $\{S_n\}$ be a random walk on the integers with negative drift, and let $A_n = \{S_k \geq 0, 1 \leq k \leq n\}$ and $A = A_\infty$. Conditioning on A is troublesome because $P(A) = 0$ and there is no natural sigma-field of events "like" A. A natural definition of $P(B|A)$ is $\lim_{n\to\infty} P(B|A_n)$. The main result here shows that this definition makes sense, at least for a large class of events B: The finite dimensional conditional distributions for the process $\{S_k\}_{k\geq 0}$ given A_n converge strongly to the finite dimensional distributions for a measure Q. This distribution Q is identified as the distribution for a stationary Markov chain on $\{0, 1, \ldots\}$.