Asymptotic Expansions in Boundary Crossing Problems

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Let S_n, $n \geq 1$ be a random walk and $t = t_a = \inf\{n \geq 1 : ng(S_n/n) > a\}$. The main results of this paper are two term asymptotic expansions as $a \to \infty$ for the marginal distributions of t_a and the normalized partial sum $S_t^* = (S_t - t\mu)/\sigma\sqrt{t}$. To leading order, S_t^* has a standard normal distribution. The effect of the randomness in the sample size t on the distribution of S_t^* appears in the correction term of the expansion.