Statistics 403 Problem Set 1

Due in lab on Friday, September 18th

1. Determine the probability distribution corresponding to the following table of cumulative probabilities.

x	0	1	2	3	4
$P(X \leq x)$	0.2	0.3	0.3	0.7	1

Solution:

Start from the left and work your way up. $P(X \leq 0) = P(X = 0) = 0.2$ (since there are no points in the sample space below zero). $P(X = 1) = P(X \leq 1) - P(X \leq 0) = 0.3 - 0.2 = 1$, and so on:

x	0	1	2	3	4
$P(X \leq x)$	0.2	0.3	0.3	0.7	1
$P(X = x)$	0.2	0.1	0	0.4	0.3

2. Suppose we have the following distribution:

x	0	1	2
$P(X = x)$	0.5	0.2	0.3

(a) What is $E(1/(X+1))$?

Solution:

$$0.5 \times 1/(0+1) + 0.2 \times 1/(1+1) + 0.3 \times 1/(2+1) = 0.7.$$

(b) What is $1/(EX+1)$?

Solution:

$$EX = 0 \times 0.5 + 1 \times 0.2 + 2 \times 0.3 = 0.8$$

so $1/(EX+1) = 1/1.8 = 5/9$.

(c) What is $E(X-2)^2$?

Solution:

$$0.5 \times (0-2)^2 + 0.2 \times (1-2)^2 + 0.3 \times (2-2)^2 = 2.2.$$

(d) What is $E(X - EX)$?

Solution:

By the additivity property, $E(X - EX) = EX - E(EX)$, and since EX is a constant, $E(EX) = EX$. Thus $E(X - EX) = 0$.

If you want to check it in this example, we know from part (b) that $EX = 0.8$, and

$$E(X - EX) = 0.5 \times (0 - 0.8) + 0.2 \times (1 - 0.8) + 0.3 \times (2 - 0.8) = 0.$$

3. For the following probability distribution, what is the probability of observing a value greater than the mean?

x	0	1	2	3	4
$P(X = x)$	0.7	0.05	0.1	0.1	0.05

Solution: The mean is $1 \times 0.05 + 2 \times 0.1 + 3 \times 0.1 + 4 \times 0.05 = 0.75$. The probability of observing a value greater than 0.75 is $0.05 + 0.1 + 0.1 + 0.05 = 0.3$.

4. Is it possible to give numerical values to a and b so that the following probability distribution has expected value 1? If so, given the values, if not, state so.

x	0	1	2	3	4
$P(X = x)$	a	0.2	0.2	0.2	b

Solution: Since the probabilities must sum to 1, b can be any number between 0 and 0.4. The value of a doesn't affect the expected value so we can ignore it. The expected value is $0.2 + 2 \times 0.2 + 3 \times 0.2 + 4 \times b = 1.2 + 4b$. Since the equation $1.2 + 4b = 1$ cannot be solved for b between 0 and 0.4, we conclude that there is no solution to the problem.

5. From the following probability distribution of X, construct the probability distribution of X^2.

x	-2	-1	0	1	2
$P(X = x)$	0.3	0.1	0.1	0.4	0.1

Solution: The sample space of X^2 is $\{0, 1, 4\}$. Since $X^2 = 0$ is equivalent to $X = 0$, it follows that $P(X^2 = 0) = P(X = 0) = 0.1$. Also, $X^2 = 1$ is equivalent to $X = 1$ or $X = -1$, so $P(X^2 = 1) = P(X = 1) + P(X = -1) = 0.4 + 0.1 = 0.5$. Similarly, $P(X^2 = 4) = P(X = 2) + P(X = -2) = 0.4$..