

Math Stat 425
Practice Exam

Instructions: Show your work and explain your reasoning carefully.

1. A committee of size three is to be selected from a group of six Democrats, five Independents, and four Republicans. What is the probability that the Democrats have a majority on the committee. What is the conditional probability that the Democrats have a majority, given that there is at least one Republican on the committee?

Ans: .341; .207

2. One card is drawn from each of six well-shuffled decks. What is the probability all four suits are represented among the six cards drawn.

Ans: .381

3. The court in Small County USA has two judges, three secretaries, and four deputies. On any given day the judges show up for work with probability .9 each, the secretaries show up with probability .6 each, and the deputies show up with probability .3 each. Court is adjourned for the day unless at least one member from each group shows up. Assuming independence, what is the probability that court is adjourned?

Ans: .296

4. A test for a rare disease has a false positive rate of 2% and a false negative rate of 1%. Suppose that .5% of the population have the disease. If a person takes the test as part of a routine physical exam, what is the probability that it will come back positive (indicating the disease to be present)? What is the conditional probability that the person has the disease, given that the test is positive?

Ans: .0249; .1992

5. Two tickets are drawn without replacement from a box containing one ticket labelled "one," two tickets labelled "two," three tickets labelled "three," and four tickets labelled "four." Let X be the absolute difference between the labels on the two tickets drawn. Find the probability mass function, mean, and standard deviation of X .

Ans: $\mu = 1.2$; $\sigma = .884$

6. On a true-false examination, a student knows the answer to any given question with probability .6 and guesses otherwise. If there are ten questions, what is probability that he/she gets at least eight of them right?

Ans: .678

7. The bottle capping machine at the ABC Brewery malfunctions with probability .0005 each time that it operates. If 10,000 bottles are produced in a day, what is the probability that there are exactly five defective caps; at least five?

Ans: .1755; .5595

8. A random variable X has a density of the form

$$f(x) = \begin{cases} c(1 - |x|) & \text{if } -1 \leq x \leq 1 \\ 0 & \text{if otherwise,} \end{cases}$$

where c is a constant. Find $P[0 < X < \frac{1}{2}]$ and the standard deviation of X .

Ans: $3/8; 1/\sqrt{6}$.

9. If X has density $f_X(x) = 2x$ for $0 < x < 1$ and $f_X(x) = 0$ otherwise, what is the density of $Y = -\log(X)$, where the natural logarithm is understood.

Ans: $f_Y(y) = 2e^{-2y}$ for $y \geq 0$.

10. A room is lighted by five globes that burn independently for exponentially distributed life times with failure rate $\lambda = .02$ per hour. What is the probability that at least one of the globes is still burning after 100 hours? What is the probability that at least three are still burning after 50 hours?

Ans: .517; .264

11. Let

$$F(x) = e^{-e^{-x}}$$

for $-\infty < x < \infty$. Show that F is a distribution function and find its median.

Ans: .3665

12. Suppose that quantitative GRE Scores are normally distributed with mean $\mu = 540$ and standard deviation $\sigma = 120$. Find the probability that a student scores more than 750. What is the 95th percentile of the distribution of scores?

Ans: .0401; 737.4