

Math/Stat 425

Problem Set 1

Due Sept. 19

Instructions: Do all of the problems. Write out the solutions to any five of the *even number problems* and turn them in on the due date. You may leave answers in terms of binomial coefficients.

1. How many four letter words can be formed if every list of four letters is a word; if each word must contain at least one vowel (a,e,i,o, or u)?

Ans: 456,976 and 262,495.

2. How many four letter words may be formed with distinct letters; distinct letters and at least one vowel?

3. How many committees of size five can be formed from 15 people if all members of the committee have the same status; if there is a chair and four others of equal status; if there is a chair, a secretary, and three others of equal status?

Ans: 3003; 15,015; and 60,060

4. How many ways can 10 students be divided into two teams of size 5 each if each team has a captain and four others of equal status and the teams are identified only by the names of their captains?

5. In how many ways can nine toys be divided among three children if the oldest child is to get 2 toys, the middle child 3, and the youngest 4?

Ans: 1260

6. In how many ways can 10 one dollar coins be divided among five children if each child is to get at least one dollar?

7. If two balanced dice are rolled, what is the probability that the sum of spots is equal to five? Determine the sample space; identify the event in question as a subset of the sample space; and compute its probability.

Ans: 1/9

8. In the previous problem, what is the probability that the absolute difference between the numbers of spots on the two dice (larger less smaller) is equal to 1?

9. What is the probability that a poker hand contains a pair (least two cards of the same denomination—i.e. at least two aces, or two two's, or ...)?

Ans: .4929

10. A drawer contains four pairs of socks—for example, a red pair, a blue pair, a black pair, and white pair? If four socks are selected at random, what is the probability that the four socks include at least one pair?

11. The *symmetric difference* between two events A and B is $A\Delta B = (A \cup B) - AB$, the event that exactly one of A or B occurs. Show that $P(A\Delta B) = P(A) + P(B) - 2P(AB)$ for any two events A and B .

12. Let A , B , and C be events and let $E = AB^cC^c \cup A^cBC^c \cup A^cB^cC$ be the event that exactly one of A , B , or C occurs. Derive an expression for $P(E)$ in terms of the probabilities of A , B , and C and their intersections.

Additional Suggested Problems

From Chapter 1: Problems 4, 7, 9, 15, 20, 24, 33.

From Chapter 2: Problems 6, 12, 15, 25, 28, 32, 34, 37.