

Math/Stat 425 Problem Set 3
Due November 5

Instructions: Do all of the problems. Write out the solutions to any five of the even numbered problems (2-12). Show your work and explain your reasoning.

1. The ABC Bolt company produces bolts of nominal diameter 1". In fact, the diameter of a typical bolt is a normally distributed random variable X with mean $\mu = 1$ " and standard deviation $\sigma = .002$ ". A bolt meets specifications if its diameter is between .997" and 1.003". What percentage of the bolts produced meet specifications?

Ans: 86.6.

2. In the Land of Oz, systolic blood pressures of college age men are normally distributed with mean $\mu = 120$ and standard deviation $\sigma = 10$. What percentage of college age men have blood pressures between 115 and 135? What percentage have blood pressure over 135?

3. Suppose that X and Y have joint probability mass function of the form $p(x, y) = c$ if x and y are integers for which $1 \leq x \leq y$ and $1 \leq y \leq 10$ and $p(x, y) = 0$ for other values of x and y , where c is a constant. Find the marginal probability mass functions of X and Y . Are X and Y independent?

Ans: $p_X(x) = (10 - x + 1)/55$ and $p_Y(y) = y/55$ for $x, y = 1, \dots, 10$. They are not independent.

4. Suppose that X and Y have joint probability mass function of the form $p(x, y) = c2^{-y}$ if x and y are integers for which $1 \leq x \leq y$ and $y \geq 1$ and $p(x, y) = 0$ for other values of x and y , where c is a constant. Find the marginal probability mass functions of X and Y and $P[X = Y]$. Are X and Y independent?

5. Let X_1 and X_2 be jointly distributed random variables with joint density function

$$f(x_1, x_2) = \frac{1}{2\pi\sqrt{(1+x_1^2+x_2^2)^3}}$$

for $-\infty < x_1, x_2 < \infty$. Find the marginal densities of X_1 and X_2 . Are X_1 and X_2 independent?

Ans: Both X_1 and X_2 have standard Cauchy distributions; they are not independent.

6. Let X_1 and X_2 be jointly distributed random variables with joint density function

$$f(x_1, x_2) = [(1 + \alpha x_1)(1 + \alpha x_2) - \alpha]e^{-x_1 - x_2 - \alpha x_1 x_2}$$

for $0 \leq x_1, x_2 < \infty$ and $f(x_1, x_2) = 0$ for other values of x_1 and x_2 , where $0 < \alpha \leq 1$. Find the marginal densities of X_1 and X_2 . Are X_1 and X_2 independent?

7. Let X and Y be independent random variables that are uniformly distributed over $[0, 1]$. Find $P\{Y \leq \sin(\pi X)\}$.

Ans: $2/\pi$.

8. Let X and Y be independent random variables that are uniformly distributed over $[0, \gamma]$, where $\gamma \geq 1$. Find the probability that the equation $z^2 + 2Xz + Y = 0$ has real roots (in z). Compute the limit of this probability as $\gamma \rightarrow \infty$.

9. Ten light globes are turned on at time $t = 0$ and allowed to burn continuously. The globes are not replaced when they burn out. Suppose that the lifetimes of the globes are independent, exponentially distributed random variables with mean $\mu = 1$ mo. Let T be the time at which the room goes dark. Find the density of T and compute $P[T \leq 3]$.

Ans: $10(1 - e^{-t})^9 e^{-t}$ for $t \geq 0$; and .600.

10. In the previous problem, suppose that lighting in the room is insufficient when two or fewer globes are burning. Let T be the time at which lighting becomes insufficient. Find the density of T and compute $P[T \leq 2]$.

11. In problem 5, find $P[0 \leq X_2 \leq 2 | X_1 = 0]$.

Ans: .447

12. In Problem 6, find $P[X_2 \leq 1 | X_1 = 1]$.

Suggested Problems. From Chapter 5: 7, 11, 15, 16, 19, 31, 32, and 40. From Chapter 6: 6,8,9,13,20,22.