Math-Stat 425
Problem Set 5
Due December 10

Instructions. Do all problems. Write out the solutions to any five of the even numbered problem. Show your work and explain your reasoning carefully.

1. The length of a man's stride is a random variable, X say, with mean $\mu = 1yd$ and standard deviation $\sigma = .1yd$. Let $X_1, \cdots, X_{100}$ denote the length of 100 strides and let $S = X_1 + \cdots + X_{100}$ denote the distance he travels in these 100 strides. Describe the approximate distribution of S and use it to approximate $P\{99 < S \le 101\}$. Find a value of $c > 0$ for which $P\{100 - c < S \le 100 + c\} \approx .95$.
Ans: .683 and $c = 1.96$.

2. Let $X_1, \cdots, X_{80}$ denote i.i.d. random variables with (common) density $f(x) = (3/4)(1-x^2)$ for $-1 < x \le 1$ and $f(x) = 0$ for other values of x, and let $S = X_1 + \cdots + X_{80}$. Describe the approximate distribution of S and use it to compute $P\{S > 3\}$. Find a value of c for which $P\{S \le c\} \approx .95$.

3. In a state lottery, a player selects a three digit number and wins 500 dollars if that number is drawn (from a box containing tickets labeled $000, \cdots, 999$. If it costs 1 to play the game, what is the player's expected gain? Now suppose that 1000 people play the game each day. Find the state's expected gain and describe its distribution.
Ans: $-.50$; 500; $1000 - 500X$, where $X \sim \text{Poisson}(\lambda = 1)$.

4. In Problem 3, let S denote the state's total gain in 100 days (assuming that 1000 people play on each day). Find $\nu := E(S)$ and a value of c for which $P\{\nu - c < S \le \nu + c\} \approx .95$.

5. Explain how to generate a pseudo random number having density $f(x) = 1/2\sqrt{x}$ for $0 < x \le 1$ and $f(x) = 0$ for other values of x.
Ans: $X = U^2$, where $U \sim \text{Unif}(0, 1]$.

6. Let X_1 and X_2 be independent random variables with density $f(x) = 1/2\sqrt{x}$ for $0 < x \le 1$ and $f(x) = 0$. Describe a simulation experiment to approximate $P\{X_1+X_2 \le 1\}$ with a standard error of at most .01.

7. Let $U \sim \text{Unif}(-\pi, \pi]$ and let $X_k = \sin(kU)$ for $k = 1, 2, \cdots$. Show that $\bar{X}_n \to^p 0$ as $n \to \infty$.

8. Let $Y_0, Y_1, Y_2, \cdots$ be independent with mean $E(Y_k) = 0$ and finite variance $E(Y_k^2) = \tau^2$ for $k = 0, 1, 2, \cdots$, and let

$$X_k = \frac{Y_{k-1} + 2Y_k + Y_{k+1}}{4}.$$

Show that $\bar{X}_n \to^p 0$ as $n \to \infty$.

9. Do Probem 22 of Chapter 8.

10. Do Problem 15 of Chapter 8.

11. Let $X_1, \cdots, X_n \sim^{\text{ind}} Unif(0,1)$ and $Y = \max[X_1, \cdots, X_n]$, the largest of the X_i. Show that Y_n converges to 1 in mean square.

Hints: First show that $E(Y_n) = n/(n+1)$ and $E(Y_n^2) = n/(n+2)$; then compute $E[(Y_n - 1)^2]$.

12. Let $X_1, \cdots, X_n \sim^{\text{ind}} \text{Exp}(\lambda = 1)$ and $Y = \min[X_1, \cdots, X_n]$, the smallest of the X_i. Show that Y_n converges to 0 in mean square.