

# Math/Stat 425

## Problem Set 1

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Note: For each problem, just one possible solution is provided. Your solutions can be different.

### 1 Solution to Question 2:

There are 26 distinct letters from A to Z. The first letter of a four letter word formed with different letters has 26 choices, the second letter has 25 choices, the third letter has 24 choices and the fourth letter has 23 choices. So there are  $26 \times 25 \times 24 \times 23 = 358800$  four letter words formed with distinct letters.

Let's first find the number of four letter words formed with different non-vowel letters. Since there are 5 vowels in the 26 letters, which are A, E, I, O, U, there are  $26 - 5 = 21$  non-vowel letters. Then similarly there are  $21 \times 20 \times 19 \times 18 = 143640$  four letter words formed with distinct non-vowel letters.

Therefore there are  $26 \times 25 \times 24 \times 23 - 21 \times 20 \times 19 \times 18 = 215160$  four letter words formed with four distinct letters and at least one vowel.

Note: If you think there are 6 vowels in the 26 letters: A, E, I, O, U and Y, then the result for the second question would be  $26 \times 25 \times 24 \times 23 - 20 \times 19 \times 18 \times 17 = 242520$ .

### 2 Solution to Question 4:

Suppose the 10 students have different names. Suppose the two teams are denoted as Team A and Team B. Then there are  $\binom{10}{1}$  choices for the leader of Team A,  $\binom{9}{1}$  choices for the leader of Team B,  $\binom{8}{4}$  for the members of Team A and  $\binom{4}{4}$  for the members of Team B. Since there is actually no denotation of Team A and Team B, the total different choices are

$$\frac{\binom{10}{1} \times \binom{9}{1} \times \binom{8}{4}}{2} = 3150.$$

### 3 Solution to Question 6:

Let's put the 10 coins in a line, then there are 9 positions between them. Now let's choose 4 positions out of the 10 ones. The coin before the 1st position will be given to the 1st child; the coin before the 2nd position will be given to the 2nd child; and so on; the coin after the 4th position will be given to the 5th child. In this way the coins are distributed as required. For those 4 positions, there are  $\binom{9}{4} = 126$  choices.

## 4 Solution to Question 8:

If two balanced dice are rolled, there are 36 possible results. Among them the following 10 ones satisfy the requirement that the absolute difference between the numbers of spots on the two dice is equal to 1:

$$\begin{aligned}(6, 5) & , (5, 6), \\ (5, 4) & , (4, 5), \\ (4, 3) & , (3, 4), \\ (3, 2) & , (2, 3), \\ (2, 1) & , (1, 2).\end{aligned}$$

Thus the probability that the absolute difference between the numbers of spots on the two dice is equal to 1 is  $\frac{10}{36}$ .

## 5 Solution to Question 10:

First let's find the probability that the four socks dose not include any pair.

If four socks are selected at random, there are  $\binom{8}{4} = 70$  possible results. Among them there are  $\binom{2}{1} \times \binom{2}{1} \times \binom{2}{1} \times \binom{2}{1} = 16$  possible results satisfying that the four socks dose not include any pair. Thus the probability that the four socks dose not include any pair is  $\frac{16}{70}$ .

Therefore the probability that the four socks include at least one pair is  $1 - \frac{16}{70} = \frac{54}{70} = \frac{27}{35}$ .

## 6 Solution to Question 11:

$$P(A \Delta B) = P(A \cup B - AB) \tag{6.0.1}$$

$$= P(A \cup B) - P(AB) \tag{6.0.2}$$

$$= P(A) + P(B) - P(AB) - P(AB) \tag{6.0.3}$$

$$= P(A) + P(B) - 2P(AB) \tag{6.0.4}$$

The following results are used:

**Result 1:** If  $A \subset B$ ,  $P(B - A) = P(B) - P(A)$ . Because  $A \cup (B - A) = B$  and  $A(B - A) = \emptyset$ ,  $P(B) = P(A \cup (B - A)) = P(A) + P(B - A)$ , which leads to the result 1. From (6.0.1) to (6.0.2), the result 1 is used.

**Result 2:**  $P(A \cup B) = P(A) + P(B) - P(AB)$ . (The proof can be found in the textbook.) From (6.0.2) to (6.0.3), the result 2 is used.

## 7 Solution to Question 12:

$$P(E) = P(AB^cC^c \cup A^cBC^c \cup A^cB^cC) \quad (7.0.5)$$

$$= P(AB^cC^c) + P(A^cBC^c) + P(A^cB^cC) \quad (7.0.6)$$

$$= P(A) - P(AB) - P(AC) + P(ABC) \quad (7.0.7)$$

$$+ P(B) - P(AB) - P(BC) + P(ABC) \quad (7.0.8)$$

$$+ P(C) - P(AC) - P(BC) + P(ABC) \quad (7.0.9)$$

$$= P(A) + P(B) + P(C) - 2(P(AB) + P(BC) + P(AC)) + 3P(ABC) \quad (7.0.10)$$

From (7.0.6) to (7.0.7-9), it is shown as follow:

$$P(AB^cC^c) = P(AB^c - AB^cC) \quad (7.0.11)$$

$$= P(AB^c) - P(AB^cC) \quad (7.0.12)$$

$$= P(A - AB) - P(AC - ABC) \quad (7.0.13)$$

$$= (P(A) - P(AB)) - (P(AC) - P(ABC)) \quad (7.0.14)$$

$$= P(A) - P(AB) - P(AC) + P(ABC) \quad (7.0.15)$$

Similarly we have

$$P(A^cBC^c) = P(B) - P(AB) - P(BC) + P(ABC)$$

and

$$P(A^cB^cC) = P(C) - P(AC) - P(BC) + P(ABC).$$

The result 1 in the Solution to Question 10 is used again and again.