STATS 520: Mathematical Methods in Statistics

Fall 2009

Summary

The aim of this course is to develop mathematical background for graduate courses in statistics and probability. The course reviews basic notions from matrix algebra and real analysis. It then introduces students to measure theory and integration. In particular, the content covers definition of measures and measurable functions, convergence theorems, Lebesgue integration, L^p spaces, signed measures, Radon–Nikodym theorem and integration on product spaces.

Prerequisite: MATH 451 or equivalent course in real analysis.

Texts: *(suggested but not required)*
James R. Schott, *Matrix Analysis for Statistics*, Wiley Series in Probability and Statistics.
W. Rudin *Principles of Mathematical Analysis*, third edition, McGraw–Hill, Inc.
H.L. Royden *Real Analysis*, third edition, Prentice Hall.

Notes: Notes on linear algebra are available on the course website. I will distribute notes on measure theory from:
M. Ross Leadbetter and Stamatis Cambanis, *Measure and Probability Theory – A basic course*

Course details:
Lectures: Mon & Wed, 2:30 pm – 4:00 pm in 271 Denninson.
Instructor: Stilian Stoev, `sstoev@umich.edu`.
Office hours: Mon & Wed, 4:15 pm – 5:45 pm in West Hall, room 445C *or* by appointment.
Credits: provides 3 credits.
Website: `http://www.stat.lsa.umich.edu/~sstoev/520/`

Grading and homework: Weekly homework will be assigned and graded. There will be one midterm and a final exam in class. The final grade will be based on the homework (30%), midterm (35%) and final (35%) exams.

No late homework will be accepted, unless extension is granted explicitly.

List of topics

Part I

- *(3 weeks)* Review of linear algebra: matrix algebra, vector and inner product spaces, orthogonal bases, quadratic forms, eigen values and eigen vectors, matrix norms and factorizations.
- *(1 week)* Basic notions from real analysis: infimum (supremum), open sets, closure, limit points, functions and their limits, differentiability, Riemann integration, change of variables, mean–value theorem. Continuity and uniform continuity, metric spaces, separability. Weierstrass approximation theorem, Cauchy sequences and completeness. Complex numbers and variables.

Part II

- *(1 week)* Classes of sets: semiring, ring, field, $\sigma-$ring, $\sigma-$field. Fields and $\sigma-$fields generated by classes of sets, Dynkin (or $\lambda-$) systems, $\pi - \lambda$ theorem. Borel sets in $\mathbb{R}$.
- *(1 week)* Measure on a semiring, extension to a ring, outer measures, extension of a measure (uniqueness in the $\sigma-$finite case). Lebesgue and Lebesgue–Stietjes measures. Completion of a measure. Approximation in measure. Measure spaces.
- *(2 weeks)* Measurable functions and transformations. Measures induced by transformations. Construction of the Lebesgue integral. Lebesgue versus Riemann integration. Lebesgue–Stietjes integrals.
- *(1–2 weeks)* Convergence of measurable functions: almost everywhere and in measure. Convergence of integrals: Fatou's lemma, Lebesgue monotone and dominated convergence theorems. Weak convergence of measures.
- *(1 week)* L^p, $(1 \leq p \leq \infty)$ spaces: completeness and separability. Inequalities: Minkowski, Hölder, Cauchy–Bunyakovski–Schwartz. The Hilbert space L^2.
- *(1 week)* Signed measures: Jordan–Hahn decomposition, absolute continuity and singularity. Radon–Nikodym theorem.
- *(1 week)* Product $\sigma-$fields and product measures: finite and countably infinite products. Fubini's theorem. Borel sets in $\mathbb{R}^d$. The space $\mathbb{R}^{\mathbb{N}}$: cylinder sets and measurability.